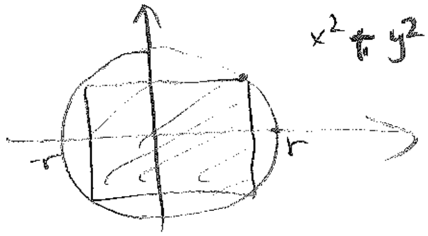
① [This problem was also in your class notes.]



$x^2 + y^2 = r^2 \qquad y = \sqrt{r^2 - x^2}$

$Q = 4xy = 4x\sqrt{r^2 - x^2}$

$Q'(x) = 4(r^2 - x^2)^{1/2} + 4x \cdot \frac{1}{2}(-2x)(r^2 - x^2)^{-1/2}$

$= \frac{4}{\sqrt{r^2 - x^2}}\left[r^2 - x^2 - x^2\right] = \frac{4(r^2 - 2x^2)}{\sqrt{r^2 - x^2}} \begin{cases} 0 \\ ? \end{cases}$

$0 \le x \le r$

$\uparrow \qquad\qquad \uparrow$

$Q(0) = 0 \qquad Q(r) = 0$

$x = \pm r$ or $x = \frac{\pm r}{\sqrt{2}}$

$Q\left(\frac{r}{\sqrt{2}}\right) = 4 \cdot \frac{r}{\sqrt{2}} \cdot \sqrt{r^2 - \frac{r^2}{2}} = \frac{4r^2}{2} = 2r^2$

By the Closed Interval Method, the maximum area occurs when $x = \frac{r}{\sqrt{2}}$ and $y = \frac{r}{\sqrt{2}}$. The dimensions of the rectangle are $\sqrt{2}r$ by $\sqrt{2}r$.

---

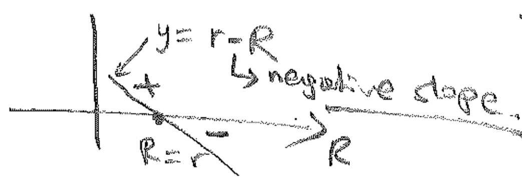
②

$P(R) = E^2 R (R+r)^{-2}$

$P'(R) = E^2 (R+r)^{-2} + E^2 R (-2)(R+r)^{-3}$

$= E^2 (R+r)^{-3}\left[R + r - 2R\right] = \frac{E^2 \cdot (r - R)}{(R+r)^3} \begin{cases} 0 \\ ? \end{cases}$

Critical # $R = r$



$(R+r)^3$ } positive.

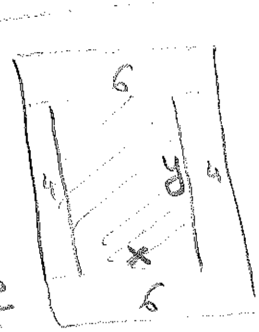
First Derivative Test

The maximum power is achieved when $R = r$. $P(r) = \frac{E^2 \cdot r}{(2r)^2}$

$P(r) = \frac{E^2}{4r}$ watts.

---

③



$x > 0$

$xy = 384$

$y = \frac{384}{x}$

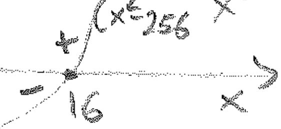
$Q = (x+8)(y+12)$

$Q(x) = (x+8)\left(\frac{384}{x} + 12\right) = 384 + 12x + 8 \cdot \frac{384}{x} + 8 \cdot 12$

$Q'(x) = 12 - \frac{8 \cdot 384}{x^2} = \frac{12(x^2 - 256)}{x^2} \begin{cases} 0 \\ ? \end{cases}$

$x = 16$

First Derivative Test.



Q is a minimum when $x = 16$ cm. / $y = 24$ cm

The dimensions of the poster are 24 by 36 cm.

① $F(\theta) = \mu W (\mu \cdot \sin\theta + \cos\theta)^{-1}$

$F'(\theta) = \mu W (-1)(\mu \cdot \sin\theta + \cos\theta)^{-2}(\mu\cos\theta - \sin\theta)$

$F'(\theta) = \dfrac{-\mu W(\mu\cos\theta - \sin\theta)}{(\mu\sin\theta + \cos\theta)^2}$ — 0, ?

$\mu\cos\theta - \sin\theta = 0$

$\dfrac{\sin\theta}{\cos\theta} = \mu$

$\tan\theta = \mu$

$\theta = \arctan\mu$

$F'(\theta)$: $-$, $+$

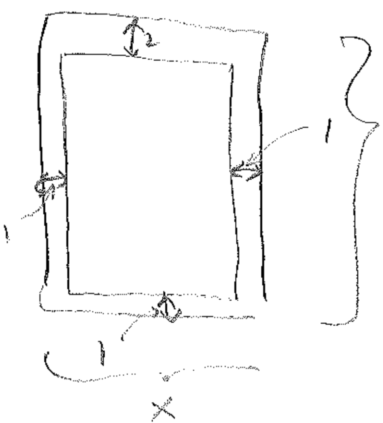
$\theta = \arctan(\mu)$

$\theta < \arctan\mu$: $\mu\cos\theta - \sin\theta > 0$

$\theta > \arctan\mu$: $\mu\cos\theta - \sin\theta < 0$

By the First Derivative Test for Global Extrema, $F(\theta)$ is at a minimum when $\theta = \arctan(\mu)$.

②



$Q = (x-2)(y-3) = (x-2)\left(\dfrac{180}{x} - 3\right)$

$xy = 180$

$y = \dfrac{180}{x}$

$= 180 - \dfrac{360}{x} - 3x + 6$

$Q(x) = 186 - 3x - \dfrac{360}{x}$

$Q'(x) = -3 + \dfrac{360}{x^2}$ — 0, ?

$= \dfrac{-3x^2 + 360}{x^2}$ — 0, ?

$x^2 = \dfrac{360}{3} = 120$

$x \neq 0$

$x = (4 \cdot 30)^{1/2}$

$x = 2\sqrt{30}$ inches

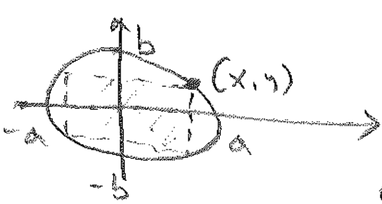
$Q'(x)$: $+$, $-$ at $2\sqrt{30}$

Sign depends only on $(360 - x^2)$.

By the first derivative test, a global max occurs when $x = 2 \cdot \sqrt{30}$ inches. $x = 2\sqrt{30}$

$y = \dfrac{180}{2\sqrt{30}} = \dfrac{3 \cdot 30}{30^{1/2}} = 3\sqrt{30}$

③



$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \Rightarrow y^2 = \left(1 - \dfrac{x^2}{a^2}\right) \cdot b^2 \Rightarrow y = \dfrac{b}{a}\sqrt{a^2 - x^2}$

$Q = 4xy = 4x \cdot \dfrac{b}{a}\sqrt{a^2 - x^2}$

$0 \le x \le a$

$Q'(x) = \dfrac{4b}{a}(a^2 - x^2)^{1/2} + \dfrac{4b}{a} x \cdot \dfrac{1}{2}(a^2 - x^2)^{-1/2} \cdot (-2x)$

$Q'(x) = \dfrac{4b}{a(a^2 - x^2)^{1/2}}\left[a^2 - x^2 - x^2\right] = \dfrac{4b}{a}\left(\dfrac{a^2 - 2x^2}{a^2 - x^2}\right)$ — 0, ?

$x = a$ or $x = \dfrac{a}{\sqrt{2}}$

$Q(0) = 0$

$Q(a) = 0$

$Q\left(\dfrac{a}{\sqrt{2}}\right) = 4 \cdot \dfrac{a}{\sqrt{2}} \cdot \dfrac{b}{a}\sqrt{a^2 - \dfrac{a^2}{2}} = \dfrac{4b}{\sqrt{2}} \cdot \dfrac{a}{\sqrt{2}} = 2ab$. [By Closed Interval Method]