Name **KEY/SHUBLEKA** **No Calculators. Present neatly. Score**_____. **G (x5) (30 mins)**
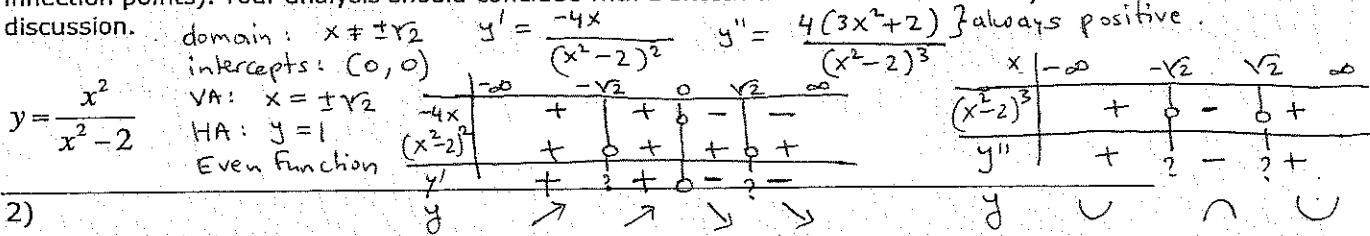
⊛ please note: this is different from $y = \dfrac{x^2}{x^2+2}$ (similar process).

1) Discuss completely (domain, intercepts, asymptotes, ID behavior, concavity, extrema, and inflection points). Your analysis should conclude with a sketch that is consistent with your discussion.

$$y = \frac{x^2}{x^2-2}$$

domain: $x \neq \pm\sqrt{2}$
intercepts: $(0,0)$
VA: $x = \pm\sqrt{2}$
HA: $y = 1$
Even function

$y' = \dfrac{-4x}{(x^2-2)^2}$

$y'' = \dfrac{4(3x^2+2)}{(x^2-2)^3}$ always positive.

	$-\infty$		$-\sqrt{2}$		0		$\sqrt{2}$		∞
$-4x$		+		+	0	−		−	
$(x^2-2)^2$		+	0	+		+	0	+	
y'		+	?	+	0	−	?	−	

x	$-\infty$		$-\sqrt{2}$		$\sqrt{2}$		∞
$(x^2-2)^3$		+	0	−	0	+	
y''		+	?	−	?	+	

| y | ↗ | ↗ | ↘ | ↘ | | y | ⌣ | ⌢ | ⌣ |

2)

A sheet of cardboard 12 in square is used to make an open box by cutting squares of equal size from the four corners and folding up the sides. What size squares should be cut to obtain a box with largest possible volume?
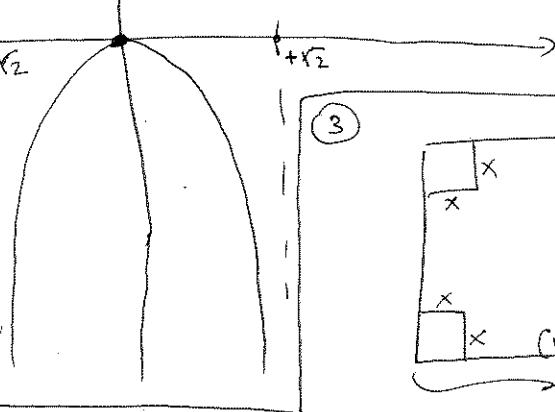
_____

3)

Let $s_A = 15t^2 + 10t + 20$ and $s_B = 5t^2 + 40t$, $t \geq 0$, be the position functions of cars A and B that are moving along parallel straight lanes of a highway.
(a) How far is car A ahead of car B when $t = 0$?
(b) At what instants of time are the cars next to each other?
(c) At what instant of time do they have the same velocity? Which car is ahead at this instant?

Your work:

(a) $s_A(0) = 20$ $s_B(0) = 0$ Car A is 20 units ahead.

(b) $s_A = s_B \to 15t^2 + 10t + 20 = 5t^2 + 40t \to$
$10t^2 - 30t + 20 = 0$
$10(t^2 - 3t + 2) = 0$
$10(t-2)(t-1) = 0$
@ $t=2$, $t=1$ the cars are next to each other.

1) Continued —

− local max @ $(0,0)$
− No inflection points
$y=1$

$-\sqrt{2}$ $+\sqrt{2}$

③

$m = 12$
$(m-2x)$
$(m-2x)$
$m = 12$

c) $V_A = 30t + 10$
$V_B = 10t + 40$ } =
$20t = 30$
$t = \dfrac{3}{2}$ seconds

$s_A(1.5) = 15 \cdot \dfrac{9}{4} + 10 \cdot \dfrac{3}{2} + 20 = 35 + \dfrac{135}{4} = \dfrac{275}{4}$

$s_B(1.5) = 5 \cdot \dfrac{9}{4} + 40 \cdot \dfrac{3}{2} = \dfrac{45}{4} + \dfrac{240}{4} = \dfrac{285}{4}$

Car B is ahead @ $t = \dfrac{3}{2}$.

$0 < x < 6$ $V(x)$

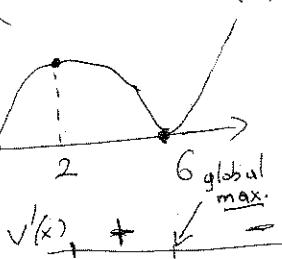$V = x(m-2x)(m-2x)$
$m = 12 \leadsto V = x(12-2x)^2$
$V = x(144 - 48x + 4x^2)$
$V = 144x - 48x^2 + 4x^3$
$V'(x) = ?$
$V'(x) = 144 - 96x + 12x^2 = 12[x^2 - 8x + 12]$
$= 12(x-2)(x-6)$

$\boxed{x = 2 \text{ inches}}$

0   $x = 2$
? $x = 6$

2 6 global max.

$V'(x)$ + − +
       2 6