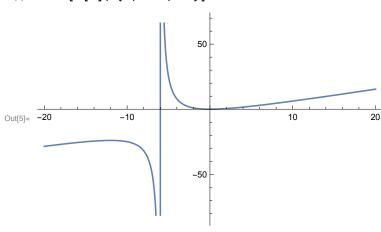
$$ln[1]:= f[x_] := x^2 / (x + 6);$$

Out[2]= 
$$\frac{x (12 + x)}{(6 + x)^2}$$

Out[3]= 
$$\frac{72}{(6+x)^3}$$



## First Derivative Test

Suppose f(x) is a continuous at x=a and the derivative is zero or undefined at x=a.

- a) If the first derivative changes from positive to negative, then f(a) is a local maximum.
- b) If the first derivative changes from negative to positive, then f(a) is a local minimum.
- c) If the first derivative does not change sign, then f(a) is not a local extremum.

## Second Derivative Test

Suppse f is twice differentiable.

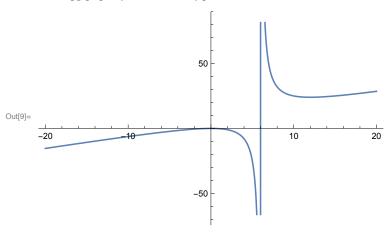
- a) If f'(a) = 0 and f''(a) > 0, then f(a) is a local minimum.
- b) If f'(a) = 0 and f''(a) < 0, then f(a) is a local maximum.
- b) If f'(a) = f''(a) = 0, then the test is inconclusive.

$$ln[6]:= g[x_] := x^2 / (x - 6);$$

Out[7]= 
$$\frac{(-12 + x) x}{(-6 + x)^2}$$

Out[8]= 
$$\frac{72}{(-6 + x)^3}$$

$$ln[9]:= Plot[g[x], \{x, -20, 20\}]$$



## The Closed Interval Method

Suppose f(x) is continuous on a closed interval [a, b]. To find global extrema:

- 1) Find critical numbers of f(x) in the open interval (a, b).
- 2) Evaluate f(a), f(b) and all the critical numbers from Step 1.
- 3) The largest and smallest values from Step 2 are the absolute maximum and minimum, respectively.