Quiz 23 KEY / SKUBLEKA

1) $\frac{d}{dx}\left[\int_0^{\pi/2} \sin\frac{x}{2}\cos\frac{x}{3}\,dx\right]$ = a number $=$ $\frac{d}{dx}(\text{constant}) = 0$

$\frac{d}{dx}\int_{x=t}^{\pi/2} \underbrace{\sin(t/2)\cos(t/3)}_{f}\,dt = \frac{d}{dx}\left(F(\pi/2) - F(x)\right) = -F'(x) = -f(x)$

$= -\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{3}\right)$ (FTC) $F' = f$

2) a) $\int_0^{\pi/4} (1+\tan t)^3 \sec^2 t\,dt = \int_1^2 u^3\,du = \frac{u^4}{4}\Big|_1^2 = \frac{16}{4} - \frac{1}{4} = \frac{15}{4}$

$u = 1 + \tan t$

$du = \sec^2 t\,dt$

b) $\int_{-\pi/4}^{\pi/4} \frac{t^4 \cdot \tan t}{2+\cos t}\,dt = 0.$ $=$ odd function / continuous on given interval $2+\cos t \neq 0$.

3) $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^9 + \left(\frac{2}{n}\right)^9 + \dots + \left(\frac{n}{n}\right)^9\right]$

$= \int_0^1 x^9\,dx = \frac{x^{10}}{10}\Big|_0^1 = \frac{1}{10}.$

$\Delta x = \frac{1-0}{n}$

$x_i = 0 + i\cdot\frac{1}{n} = \frac{i}{n}$

$f(x_i) = (x_i)^9 \rightsquigarrow f(x) = x^9$

4) $\int_0^2 f(x)\,dx = 6$

$\int_0^{\pi/2} f(2\sin\theta)\cos\theta\,d\theta = \frac{1}{2}\int_0^{\pi/2} f(\underbrace{2\sin\theta}_{u})\cdot\underbrace{2\cos\theta\,d\theta}_{du}$

$= \frac{1}{2}\int_0^2 f(u)\,du = \frac{1}{2}\cdot 6 = 3.$

5) $\lim_{h\to 0}\frac{1}{h}\int_2^{2+h} \underbrace{\sqrt{1+t^3}}_{f(t)}\,dt = \lim_{h\to 0}\frac{1}{h}\left(F(2+h) - F(2)\right)$ $F' = f$

$= \lim_{h\to 0}\frac{F(2+h)-F(2)}{h} = F'(2) = f(2) = \sqrt{1+2^3} = 3.$