

Quiz 23 KEY / SHUBLEKA

$$1) \frac{d}{dx} \int_0^{\pi/2} \sin \frac{x}{2} \cos \frac{x}{3} dx = \text{a number} \\ = \frac{d}{dx} (\text{constant}) = 0$$

$$\frac{d}{dx} \int_{x=t}^{\pi/2=t} \underbrace{\sin(t/2) \cos(t/3)}_f dt = \frac{d}{dx} (F(\pi/2) - F(x)) = -F'(x) = -f(x) \\ = -\sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{3}\right) \quad \begin{matrix} \text{(FTC)} \\ F' = f \end{matrix}$$

$$2) a) \int_0^{\pi/4} (1 + \tan t)^3 \sec^2 t dt = \int_1^2 u^3 du = \left. \frac{u^4}{4} \right|_1^2 = \frac{16}{4} - \frac{1}{4} = \frac{15}{4}$$

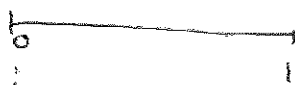
$u = 1 + \tan t$
 $du = \sec^2 t dt$

$$b) \int_{-\pi/4}^{\pi/4} \frac{t^4 \cdot \tan t}{2 + \cos t} dt = 0$$

= odd function / continuous on given interval & cost ≠ 0.

$$3) \lim_{n \rightarrow \infty} \frac{1}{n} \left[\left(\frac{1}{n}\right)^9 + \left(\frac{2}{n}\right)^9 + \dots + \left(\frac{n}{n}\right)^9 \right]$$

$\Delta x = \frac{1-0}{n}$



$$= \int_0^1 x^9 dx = \left. \frac{x^{10}}{10} \right|_0^1 = \frac{1}{10}$$

$x_i = 0 + i \cdot \frac{1}{n} = \frac{i}{n}$
 $f(x_i) = \left(\frac{i}{n}\right)^9 \rightsquigarrow f(x) = x^9$

$$4) \int_0^2 f(x) dx = 6 \quad \int_0^{\pi/2} f(2 \sin \theta) \cos \theta d\theta = \frac{1}{2} \int_0^{\pi/2} \underbrace{f(2 \sin \theta)}_u \cdot \underbrace{2 \cos \theta d\theta}_{du}$$

$$= \frac{1}{2} \int_0^2 f(u) du = \frac{1}{2} \cdot 6 = 3$$

$$5) \lim_{h \rightarrow 0} \frac{1}{h} \int_2^{2+h} \underbrace{\sqrt{1+t^3}}_{f(t)} dt = \lim_{h \rightarrow 0} \frac{1}{h} (F(2+h) - F(2)) \quad F' = f$$

$$= \lim_{h \rightarrow 0} \frac{F(2+h) - F(2)}{h} = F'(2) = f(2) = \sqrt{1+2^3} = 3$$