12/04/2015
D. Shubleka

Course: AP Calculus BC | Quiz: 22 | Instructor: D. Shubleka

**Name__________ No calculators. Present neatly. Score___.**

1)

Find the shortest distance from a fixed point $(x_1, y_1)$ to the straight line $Ax + By = C$.

2)

Use limits to find the area that graph of $f(x) = x^2 + 2x + 3$ forms with the horizontal axis on the interval [-1, 3]. (Show all the work leading to your answer.)

Your work:

1) $(x_1, y_1) \longleftrightarrow \left(x, \frac{C-Ax}{B}\right)$

$$Q = D^2 = (x - x_1)^2 + \left(\frac{C}{B} - \frac{A}{B}x - y_1\right)^2$$

$$Q' = 2(x - x_1) + 2\left(\frac{C}{B} - y_1 - \frac{A}{B}x\right)\cdot\left(\frac{-A}{B}\right)$$

$$Q' = 2x - 2x_1 - \frac{2AC}{B^2} + \frac{2Ay_1}{B} + 2\frac{A^2}{B^2}x <^{0}_{?}$$ (never undefined)

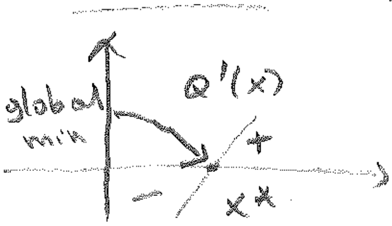
$$\left(2 + 2\frac{A^2}{B^2}\right)x = \frac{2AC}{B^2} + 2x_1 - 2\frac{Ay_1}{B}$$

$$B^2\left[\left(1 + \frac{A^2}{B^2}\right)x\right] = \left[\frac{AC}{B^2} + x_1 - \frac{Ay_1}{B}\right]\cdot B^2$$

$$(A^2 + B^2)x = AC + B^2x_1 - ABy_1$$

$$x^* = \frac{AC + B^2x_1 - ABy_1}{A^2 + B^2}$$

Justification: $Q(x) = \underbrace{\left(2 + 2\frac{A^2}{B^2}\right)}_{\text{positive slope}} x \underbrace{- 2x_1 - \frac{2AC}{B^2} + \frac{2Ay_1}{B}}_{\text{y int}}$

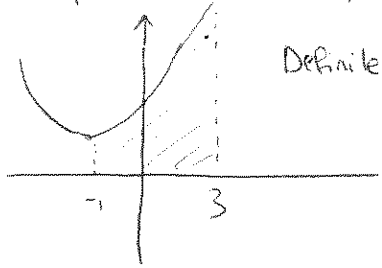



$$D = \sqrt{\left(\frac{AC + B^2x_1 - ABy_1}{A^2 + B^2} - x_1\right)^2 + \left(\frac{C}{B} - y_1 - \frac{A}{B}\cdot\frac{AC + B^2x_1 - ABy_1}{A^2 + B^2} - y_1\right)^2}$$

Note: this can be simplified

② $f(x) = x^2 + 2x + 3 \qquad [-1, 3]$

$f(x) = x^2 + 2x + 1 + 2 = (x+1)^2 + 1 > 0$ for all $x$.




Definite Integral $=$ Area in this case.

$$\Delta x = \frac{b-a}{n} = \frac{3-(-1)}{n} = \frac{4}{n}$$

$$x_i = a + i \cdot \Delta x = -1 + \frac{4i}{n}$$

$$f(x_i) = \left(\frac{4i}{n} - 1\right)^2 + 2\left(\frac{4i}{n} - 1\right) + 3 = \frac{16 i^2}{n^2} - \frac{8i}{n} + 1 + \frac{8i}{n} - 2 + 3 = \frac{16}{n^2} i^2 + 2$$

$$\int_{-1}^{3} f(x)\, dx = \int_{-1}^{3} x^2 + 2x + 3\, dx = \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{16}{n^2} \cdot i^2 + 2\right) \cdot \frac{4}{n}$$

$$= \lim_{n \to \infty} \frac{4}{n} \left[ \sum_{i=1}^{n} \frac{16}{n^2} i^2 + \sum_{i=1}^{n} 2 \right] = \lim_{n \to \infty} \frac{4}{n} \left[ \frac{16}{n^2} \sum i^2 + 2n \right]$$

$$= \lim_{n \to \infty} \left[ \frac{4}{n} \left( \frac{16}{n^2} \frac{n(n+1)(2n+1)}{6} + 2n \right) \right] = \lim_{n \to \infty} \left[ \frac{128 n^3 + \dots}{6 n^3} + \frac{8n}{n} \right]$$

$$= \frac{64}{3} + 8 = \frac{64}{3} + \frac{24}{3} = \frac{88}{3}$$

12/04/2015
D. Shubleka



**Name** ~~KEY~~ **No calculators. Present neatly. Score___.**

1)

Find the shortest distance from a fixed point $(x_1, y_1)$ to the straight line $Ax + By = C$.

2)

Use limits to find the area that graph of $f(x) = x^3 + x^2 + 3$ forms with the horizontal axis on the interval [-1, 3]. (Show all the work leading to your answer.)

Your work:

2) $f(x) = x^3 + x^2 + 3$ $\quad f'(x) = 3x^2 + 2x = x(3x+2) < 0$ on $\left(\frac{-2}{3}; 0\right)$

$f(-1) = 3$

$\begin{cases} f(\frac{-2}{3}) > 0 \\ f(0) > 0 \end{cases}$

$\Delta x = \frac{3-(-1)}{n} = \frac{4}{n}$

$x_i = -1 + \frac{4}{n} i = \frac{4i}{n} - 1$

$f(x_i) = \left(\frac{4i}{n} - 1\right)^3 + \left(\frac{4i}{n} - 1\right)^2 + 3 = \frac{64}{n^3} i^3 - \frac{48 i^2}{n^2} + \frac{12 i}{n} - 1 + \frac{16 i^2}{n^2} - \frac{8i}{n} + 1 + 3$

$f(x_i) = \frac{64}{n^3} i^3 - \frac{32}{n^2} i^2 + \frac{4}{n} i + 3$

$\int_{-1}^{3} f(x)\,dx = \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \left(\frac{64}{n^3} i^3 - \frac{32}{n^2} i^2 + \frac{4}{n} i + 3\right)$

$= \lim_{n\to\infty} \frac{4}{n} \left[\frac{64}{n^3} \sum i^3 - \frac{32}{n^2} \sum i^2 + \frac{4}{n} \sum i + 3n\right]$

$= \lim_{n\to\infty} \frac{4}{n} \left[\frac{64}{n^3} \cdot \left(\frac{n(n+1)}{2}\right)^2 - \frac{32}{n^2} \frac{n(n+1)(2n+1)}{6} + \frac{4}{n} \frac{n(n+1)}{2} + 3n\right]$

$= \lim_{n\to\infty} \left(\frac{256 n^4 + \ldots}{4 n^4} - \frac{256 n^3 + \ldots}{6 n^3} + \frac{16 n^2 + \ldots}{2n^2} + \frac{12 n}{n}\right)$

$= 64 - \frac{128}{3} + 8 + 12 = 84 - \frac{128}{3} = \boxed{\frac{124}{3}}$