Name_____ No calculators. Present neatly. Score_____. B (10 min)

1)

Suppose that f is continuous on $[0, 4], f(0) = 1$, and $2 \leq f'(x) \leq 5$ for all x in $(0, 4)$. Show that $9 \leq f(4) \leq 21$.

2)

Show that the equation $3x + 2\cos x + 5 = 0$ has exactly one real root.

Your work:

1) Apply the Mean Value Theorem on [0, 4].

$$\frac{f(4)-f(0)}{4-0} = f'(c)$$

$$2 \leq \frac{f(4)-1}{4} \leq 5$$

$$8 \leq f(4)-1 \leq 20$$

$$9 \leq f(4) \leq 21$$

2)

$f(x) = 3x + 2\cos x + 5$ is continuous and differentiable everywhere.

Use IVT to show that a sign change (f(-3) vs f(0)) implies at least one root.

Argue by contradiction that f(x) cannot have two zeroes. If it does, we can apply Rolle's Theorem on the zeroes' interval. The first derivative is never equal to zero, however.