Name KEY/SHUBLEKA **No calculators. Present neatly. Score________.**

1) Brandon is on one side of a river that is 50 m wide and wants to reach a point 200 m downstream on the opposite side as quickly as possible by swimming diagonally across the river and then running the rest of the way. Find the best route if Brandon can swim at 1.5 m/s and run at 4 m/s.

2) Find the transition points, intervals of increase/decrease, concavity, and asymptotic behavior. Then sketch the graph, with this information indicated.

$$y=(2x^2-1)e^{-x^2}$$

3) Briefly describe Newton's Method. Include its definition and use. $x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$

Newton's Method is used to approximate roots of differentiable functions.

Your work:

1) see other copy.

2) $y=(2x^2-1)\cdot e^{-x^2}$ y-intercept: $(0,-1)$ even function $\lim_{x\to\infty} f(x)=0$

x-intercepts: $2x^2=1$, $x=\pm\sqrt{2}/2$ $\lim_{x\to-\infty} f(x)=0$

$y'=(2x^2-1)\cdot(-2x)\,e^{-x^2}+4x\cdot e^{-x^2}$

$=\left[-4x^3+2x+4x\right]e^{-x^2}=-2x\left[2x^2-3\right]e^{-x^2}$

$y''=\frac{d}{dx}\left[(-4x^3+6x)\cdot e^{-x^2}\right]=(-12x^2+6)\cdot e^{-x^2}+(-4x^3+6x)\cdot(-2x)\cdot e^{-x^2}$

$=e^{-x^2}\left[-12x^2+6+8x^4-12x^2\right]=e^{-x^2}\left[8x^4-24x^2+6\right]$

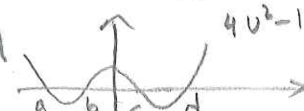
$=e^{-x^2}\cdot 2\left[4x^4-12x^2+3\right]=2e^{-x^2}$

$=e^{-x^2}\cdot 2\left[4u^2-12u+3\right]$

$u=\frac{12\pm\sqrt{144-48}}{8}=\frac{12\pm4\sqrt6}{8}=\frac{3\pm\sqrt6}{2}$

$x=\pm\sqrt{\frac{3\pm\sqrt6}{2}}$ ⇝ a, b, c, d

$4u^2-12u+3=y$

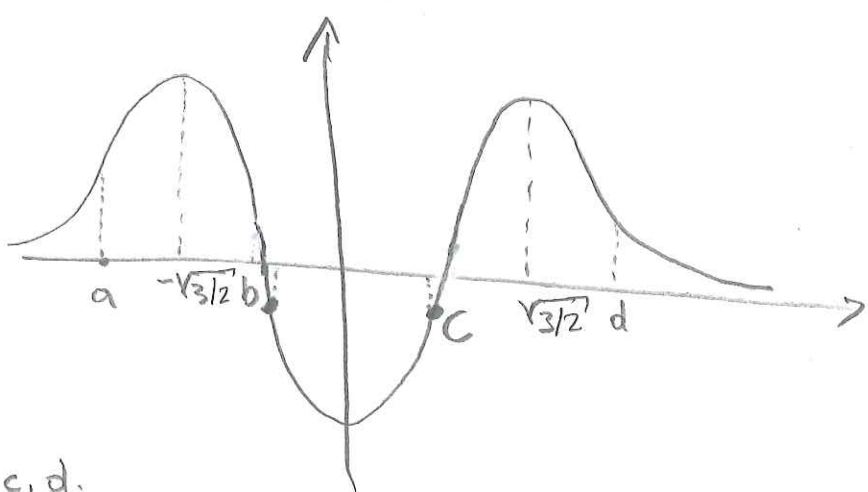


y' x	$-\infty$	$-\sqrt{3/2}$		0		$\sqrt{3/2}$	∞
$-2x$	+		+	0	−		−
x^2-3	+	0	−		−	0	+
y'	+	0	−	0	+	0	−
y	↗		↘		↗		↘

y'' x	$-\infty$	a		b		c		d	∞
y''	+	0	−	0	+	0	−	0	+
y	∪		∩		∪		∩		∪

rel max @ $x=\pm\sqrt{3/2}$ Inflection points @ a, b, c, d.

rel min @ $x=0$



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2) Find the transition points, intervals of increase/decrease, concavity, and asymptotic behavior. Then sketch the graph, with this information indicated.

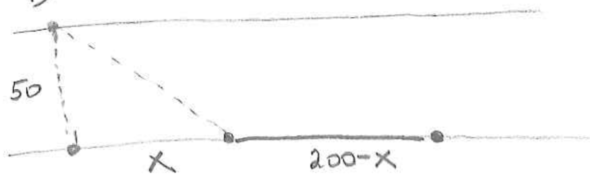
$y = xe^{-x^2}$

3) Briefly describe the Mean Value Theorem. Include its definition and use.

If $f(x)$ is cont. on $[a,b]$ and differentiable on (a,b), then at least once in (a,b) $f'(c) = \frac{f(b)-f(a)}{b-a}$, $a<c<b$. When $a=b$, we call it Rolle's Theorem. We use it to find points where instantaneous rate is equal to avg. rate.

Your work

(1)



$0 \le x \le 200$

$Q = T_{SWIM} + T_{RUN}$

$Q(x) = \frac{\sqrt{x^2+2500}}{1.5} + \frac{200-x}{4}$

$Q(x) = \frac{2}{3}(x^2+2500)^{1/2} - \frac{1}{4}x + 50$

$Q'(x) = \frac{1}{3}(x^2+2500)^{-1/2} \cdot 2x - \frac{1}{4}$

$Q'(x) = \frac{2x}{3\sqrt{x^2+2500}} - \frac{1}{4} = \frac{8x - 3\sqrt{x^2+2500}}{12\sqrt{x^2+2500}}$ — 0; ? (none)

$8x = 3\sqrt{x^2+2500}$

$64x^2 = 9x^2 + 9 \cdot 2500$

$55x^2 = 9 \cdot 2500$

$x = \frac{150}{\sqrt{55}}$ m ≈ 20.226 m

$Q(0) = \frac{50}{1.5} + 50 = 83.333$ s

$Q(200) = 137.437$ s

$Q\left(\frac{150}{\sqrt{55}}\right) = 80.901$ s ←

By Closed Interval Method should swim to a point $\approx$ 20.226 m downstream, and run the rest.

(2) $y = xe^{-x^2}$ odd function

$y' = e^{-x^2} - 2x^2 \cdot e^{-x^2} = e^{-x^2}(1-2x^2)$

$y'' = -2xe^{-x^2} - 4xe^{-x^2} + 4x^3e^{-x^2} = e^{-x^2} \cdot 2x[-1-2+2x^2]$

$y'' = -2xe^{-x^2}[3-2x^2]$

y'

x	$-\infty$		$-\sqrt{2}/2$		$\sqrt{2}/2$		∞
$1-2x^2$		−	0	+	0	−	
y'		−	0	+	0	−	
y		↘		↗		↘	

Rel. min @ $x = -\frac{\sqrt{2}}{2}$

Rel. max @ $x = \frac{\sqrt{2}}{2}$

intercepts: $(0,0)$

$\lim_{x\to\infty} f(x) = 0$

$\lim_{x\to-\infty} f(x) = 0$

y''

x	$-\infty$		$-\sqrt{\frac{3}{2}}$		0		$\sqrt{\frac{3}{2}}$		∞
$-2x$		+		+	0	−		−	
$3-2x^2$		−	0	+		+	0	−	
y''		−	0	+	0	−	0	+	
y		∩		∪		∩		∪	

inflection points