Name KEY/SHUBLEKA. **No Calculators. Present neatly. Score________.**

1.

The radius of a right circular cylinder is increasing at a rate of 2 *in/min* and the height is decreasing at a rate of 3 *in/min*. At what rate is the volume changing when the radius is 8 *in* and the height is 12 *in*? Is the volume increasing or decreasing?

2.

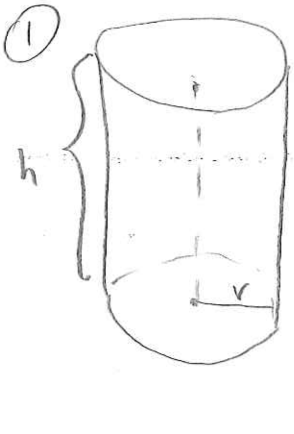
A train, starting at 11am, travels east at 45 *mph* while another, starting at noon from the same point, travels south at 60 *mph*. How fast are they separating at 3pm?

3.

Find a point on the graph of $y = e^{3x}$ at which the tangent line passes through the origin.

Your work:

①



$V = \pi r^2 h$ $\quad \frac{dh}{dt} = -3$ in/min $\quad \frac{dr}{dt} = 2$ in/min

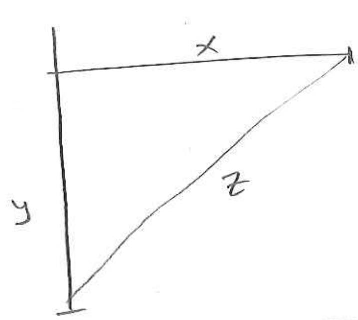
$$\frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt} + 2\pi r \cdot h \cdot \frac{dr}{dt}$$

@ $r = 8, h = 12$:

$$\frac{dV}{dt} = \pi \cdot 64 \cdot (-3) + 2\pi \cdot 8 \cdot 12 \cdot 2$$
$$= -3 \cdot 64\pi + 6 \cdot 64\pi$$
$$= 3 \cdot 64\pi = 192\pi \text{ in}^3/\text{min}$$

The volume is increasing at a rate of of 192π in³/min when r= 8 in and h= 12 in.

②



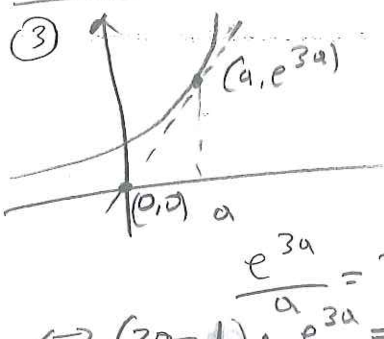
$\frac{dx}{dt} = 45$ mph $\quad x(3PM) = 180$ miles

$\frac{dy}{dt} = 60$ mph $\quad y(3PM) = 180$ miles

$$z^2 = x^2 + y^2$$
$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2 \cdot y \cdot dy/dt$$
$$\frac{dz}{dt} = \frac{180 \cdot 45 + 180 \cdot 60}{\sqrt{180^2 + 180^2}}$$
$$= \frac{60 \cdot 3 \cdot 45 + 60 \cdot 3 \cdot 60}{60 \cdot \sqrt{3^2 + 3^2}}$$
$$= \frac{3 \cdot 45 + 60 \cdot 3}{3\sqrt{2}} = \frac{105}{\sqrt{2}} \text{ mph}$$
$$\approx 74.246 \text{ mph}$$

At 3pm the trains are separating @ 74.25mph.

③



$$\frac{dy}{dx}\Big|_{x=a} = 3 \cdot e^{3a}$$

$$\text{slope} = \frac{e^{3a} - 0}{a - 0}$$

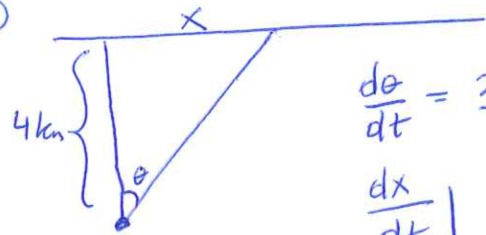
$$\frac{e^{3a}}{a} = 3e^{3a}$$

$$\Leftrightarrow (3a - 1) \cdot e^{3a} = 0 \Rightarrow a = \frac{1}{3}$$

$$\left(\frac{1}{3}, e\right)$$

Quiz 19 | AP Calculus AB | Solutions | Shubleka.

①



$$\frac{d\theta}{dt} = \frac{2\pi}{10} = \pi/5 \text{ rad/s}$$

$$\left.\frac{dx}{dt}\right|_{\theta=\pi/4} = ?$$

$$\tan\theta = \frac{x}{4}$$

$$4\tan\theta = x$$

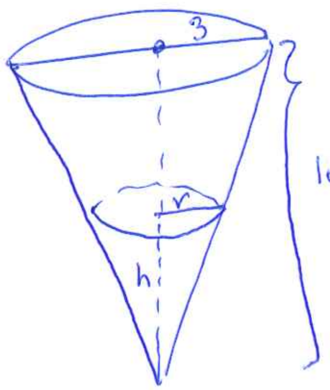
$$4\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{dx}{dt}$$

$$4 \cdot \sec^2\frac{\pi}{4} \cdot \frac{d\theta}{dt} = \frac{dx}{dt}$$

$$4 \cdot 2 \cdot \frac{\pi}{5} = \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{8\pi}{5} \text{ km/s}.$$

②



$$\frac{dV}{dt} = -4 \text{ ft}^3/\text{min}$$

$$\frac{r}{h} = \frac{3}{10} \Rightarrow r = \frac{3h}{10}$$

$$V = \frac{1}{3}\pi r^2 \cdot h \Rightarrow V = \frac{1}{3} \cdot \pi \cdot \frac{9h^2}{100} \cdot h$$

$$\frac{dh}{dt} = ?$$

$$V = \frac{3\pi}{100} \cdot h^3$$

$$\frac{dV}{dt} = \frac{9\pi h^2}{100} \cdot \frac{dh}{dt}$$

@ $h = 6$ ft. : $-4 = \frac{9\pi}{100} \cdot 36 \cdot \frac{dh}{dt}$

$$\frac{dh}{dt} = \frac{-100}{81\pi} \text{ ft/min}$$

$$\frac{dA}{dt} = \frac{d}{dt}(\pi r^2) = $$
$$= 2\pi r \cdot \frac{dr}{dt}$$
$$= 2\pi \cdot \frac{18}{10} \cdot \left(\frac{-10}{27\pi}\right)$$
$$= \frac{-4}{3} \frac{\text{ft}^2}{\text{min}}$$

③

y $\underline{\text{tangent}}$ $y = x$

$$\frac{dy}{dx} = \frac{1}{x \cdot \ln b} = 1 \Rightarrow x = \frac{1}{\ln b}$$

and

$$\log_b x = x \Rightarrow x = b^x$$

$$\frac{\ln x}{\cancel{\ln b}} = x = \frac{1}{\cancel{\ln b}} \Rightarrow \ln x = 1 \Rightarrow \boxed{x = e}$$

$b = ?$

$$\ln b = \frac{1}{e}$$

$$\boxed{b = e^{1/e} = \sqrt[e]{e}}$$