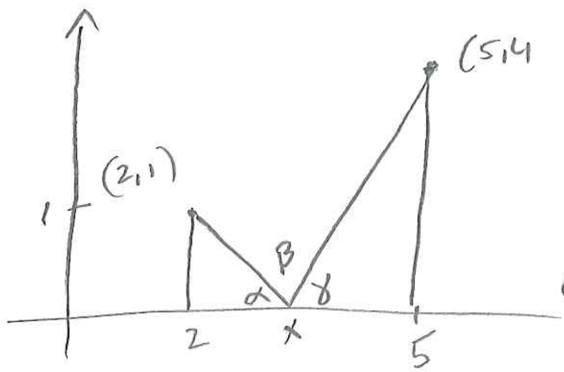
Course: AP Calculus BC | Quiz: 18 | Instructor: D. Shubleka

Name KEY / SHUBLEKA **No calculators. Present neatly. Score________. C**

1) Given points A(2, 1) and B(5, 4), find the point P in the interval [2, 5] on the x-axis that maximizes angle *APB*. Your work:

Your work



$$\tan\alpha = \frac{1}{x-2}$$

$$\tan\gamma = \frac{4}{5-x}$$

$$Q = \beta = \pi - \arctan\left(\frac{1}{x-2}\right) - \arctan\left(\frac{4}{5-x}\right)$$

$$Q'(x) = -\frac{1}{1+\left(\frac{1}{x-2}\right)^2}\cdot\frac{-1}{(x-2)^2}$$

$$-\frac{1}{1+\left(\frac{4}{5-x}\right)^2}\cdot(-4)\cdot(5-x)^{-2}\cdot(-1)$$

$$Q'(x) = \frac{1}{(x-2)^2+1} - 4\,\frac{1}{(5-x)^2+16}$$

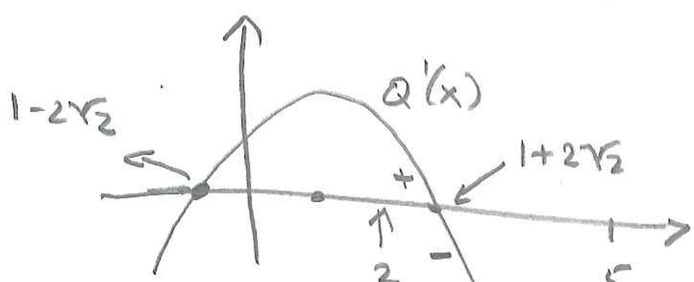
$$= \frac{(5-x)^2+16-4(x-2)^2-4}{\underbrace{\left[(x-2)^2+1\right]}\ \underbrace{\left[(5-x)^2+16\right]}} \quad \begin{matrix} > 0 \\ < ? \end{matrix}$$

always positive

$$25-10x+x^2+\cancel{16}$$
$$-4x^2+16x-\cancel{16}-4$$
$$= -3x^2+6x+21$$
$$= -3\left[x^2-2x-7\right]$$

$$x = \frac{2\pm\sqrt{4+28}}{2}$$

$$x = \frac{2\pm\sqrt{32}}{2} = 1\pm 2\sqrt{2}$$



By the First Derivative Test for Global Extrema, the angle is maximized when $x = 1+2\sqrt{2}$. $P(1+2\sqrt{2}, 0)$.