Course: AP Calculus BC | Quiz: 17 | Instructor: D. Shubleka

Name SHUBLEKA / KEY **No calculators. Present neatly. Score________.**

1) Produce the graph of f that reveals all the important aspects of the curve. In particular, you should use the signs of f' and f'' to identify the intervals of increase and decrease, extreme values, intervals of concavity, and inflection points.

symmetry: odd function!!

$y = 2x - \tan x, \quad -\pi/2 < x < \pi/2$

Your work:

$f'(x) = 2 - \sec^2 x = 2 - \dfrac{1}{\cos^2 x} = \dfrac{2\cos^2 x - 1}{\cos^2 x}$ ← always positive on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$f'(x) =$ 0, ?

$2\cos^2 x = 1 \Rightarrow \cos x = \pm\dfrac{\sqrt{2}}{2}$

$x = \pm\dfrac{\pi}{4}$

x	$-\pi/2$		$-\pi/4$		$\pi/4$		$\pi/2$
$\sqrt{2}\cos x - 1$	?	$-$	0	$+$	0	$-$	?
$\sqrt{2}\cos x + 1$	?	$+$		$+$		$+$	?
$\cos^2 x$	?						?
$f'(x)$	?	$-$	0	$+$	0	$-$	?
$f(x)$		↘		↗		↘	

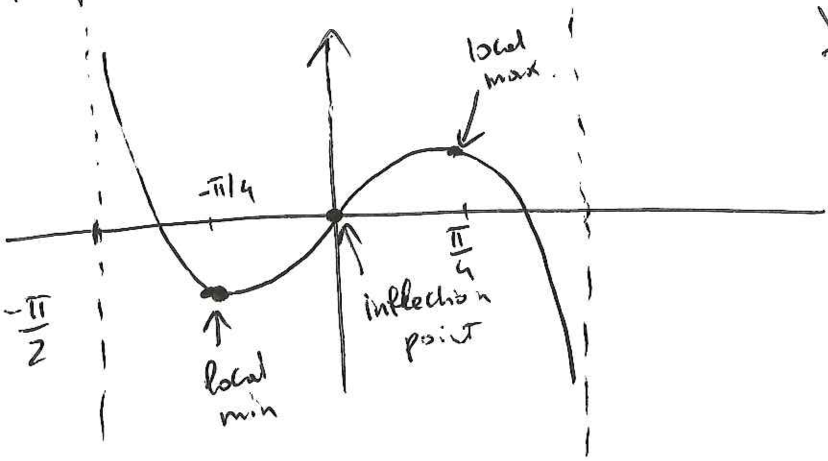
local max: $\left(\dfrac{\pi}{4}, \dfrac{\pi}{2} - 1\right)$

local min: $\left(\dfrac{-\pi}{4}, \dfrac{-\pi}{2} + 1\right)$

$f''(x) = -2\sec x \cdot \sec x \tan x = -2\underbrace{\sec^2 x}_{\text{positive}} \tan x$

x	$-\pi/2$		0		$\pi/2$
$-2\tan x$		$+$	0	$-$	
$f''(x)$		$+$	0	$-$	
$f(x)$		∪		∩	

inflection point @ $(0, 0)$



VA: $x = \dfrac{\pi}{2}$

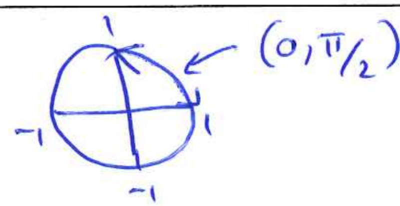
no global extrema.

Name KEY / SHUBLEKA **No calculators. Present neatly. Score________. C**

1) Produce the graph of f that reveals all the important aspects of the curve. In particular, you should use the signs of f' and f'' to identify the intervals of increase and decrease, extreme values, intervals of concavity, and inflection points.

$$y = \sec x + \tan x, \quad 0 < x < \pi/2$$

Your work:



$f(x) = \sec x + \tan x$

$f'(x) = \sec x \tan x + \sec^2 x$

$= \underbrace{\sec x}_{\text{positive}} \left[\underbrace{\tan x + \sec x}_{\text{positive}} \right]$ in Quadrant I.

$f'(x) > 0$ on $(0, \pi/2)$, therefore $f(x)$ is increasing on $(0, \pi/2)$ and there are no local extrema.

$f''(x) = (\sec x \tan x) \tan x + \sec x \sec^2 x + 2 \sec x \cdot (\sec x \tan x)$

$f''(x) = \sec x \left[\tan^2 x + 2 \tan x \sec x + \sec^2 x \right] =$

$= \underbrace{\sec x}_{\text{positive}} \underbrace{(\tan x + \sec x)^2}_{\text{positive}} > 0$

$f''(x) > 0$ on $(0, \pi/2)$, therefore $f(x)$ is concave up on $(0, \pi/2)$ and there are no inflection points.

$\left.\begin{array}{l} f(0) = ? \\ f(\pi/2) = ? \end{array}\right\}$ undefined, but: $\lim_{x \to 0^+} f(x) = 1 + 0 = 1$

$\lim_{x \to \frac{\pi}{2}^-} f(x) = 0 + \infty = \infty$ & VA @ $x = \frac{\pi}{2}$