Name___________ No Calculators. Present neatly. Score________.

1.

Find the coordinates of the point in the first quadrant at which the tangent line to the curve $x^3 - xy + y^3 = 0$ is parallel to the x-axis.

Your work:

$$x^3 - xy + y^3 = 0$$

$$3x^2 - y - x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(3y^2 - x) = y - 3x^2 \rightarrow \frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$$

$$slope = 0$$

$$\frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x} \rightarrow y - 3x^2 = 0 \rightarrow y = 3x^2$$

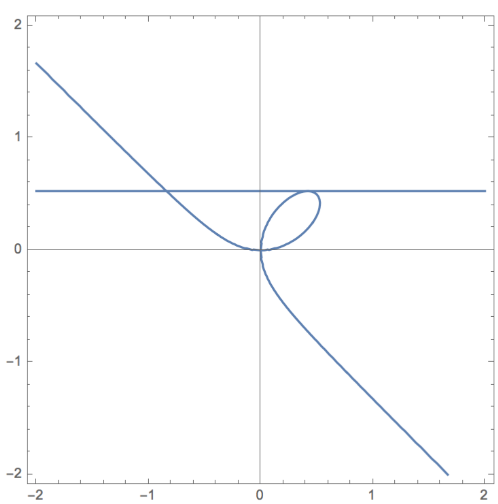
Plug this into the original equation:

$$x^3 - x\left(3x^2\right) + \left(3x^2\right)^3 = 0 \rightarrow -2x^3 + 27x^6 = 0 \rightarrow x^3(27x^3 - 2) = 0$$

$$x = 0; x = \frac{2^{1/3}}{3} \rightarrow y = 0; y = \frac{2^{2/3}}{3}$$

$$\left(\frac{2^{1/3}}{3}, \frac{2^{2/3}}{3}\right)$$

$$Note: x > 0, y > 0$$



Name___________ No Calculators. Present neatly. Score______.

1.

Find the coordinates of the point in the first quadrant at which the tangent line to the curve $x^3 - xy + y^3 = 0$ is parallel to the y-axis.

Your work:

$$x^3 - xy + y^3 = 0$$

$$3x^2 - y - x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(3y^2 - x) = y - 3x^2 \rightarrow \frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$$

$$slope = undefined$$

$$\frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x} \rightarrow 3y^2 - x = 0 \rightarrow x = 3y^2$$

Plug this into the original equation:

$$\left(3y^2\right)^3 - \left(3y^2\right)y + y^3 = 0 \rightarrow 27y^6 - 2y^3 = 0 \rightarrow y^3(27y^3 - 2) = 0$$

$$y = 0; y = \frac{2^{1/3}}{3} \rightarrow x = 0; x = \frac{2^{2/3}}{3}$$

$$\left(\frac{2^{2/3}}{3}, \frac{2^{1/3}}{3}\right)$$

$$Note: x > 0, y > 0$$