

Name _____ No Calculators. Present neatly. Score _____.

1. Find equations for two lines through the origin that are tangent to the ellipse

$$2x^2 - 4x + y^2 + 1 = 0$$

Your work:

$$2x^2 - 4x + y^2 + 1 = 0$$

$$4x - 4 + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{4 - 4x}{2y} = \frac{2 - 2x}{y}$$

We can also write slope as rise over run from $(0, 0)$ to the point where the tangent line meets the ellipse:

$$\text{slope} = \frac{y_1 - y_0}{x_1 - x_0} = \frac{y_1 - 0}{x_1 - 0} = \frac{y_1}{x_1} = \frac{2 - 2x_1}{y_1}$$

Cross multiply, to get:

$$y_1^2 = 2x_1 - 2x_1^2$$

Plug this into the original equation of the ellipse:

$$2x_1^2 - 4x_1 + y_1^2 + 1 = 0 \rightarrow 2x_1^2 - 4x_1 + (2x_1 - 2x_1^2) + 1 = 0$$

$$\rightarrow x_1 = \frac{1}{2}$$

$$\rightarrow y_1 = \pm \frac{\sqrt{2}}{2}$$

$$\text{slope} = \frac{y_1}{x_1} = \pm \sqrt{2}$$

$$y = \sqrt{2}x$$

$$y = -\sqrt{2}x$$