Name KEY **No Calculators. Present neatly. Score________. A (10min)**

1.Use the limit definition to find the derivative of f(x).

$$f(x) = \frac{4-x}{3+x}$$

2. At what point(s) is the tangent line to the curve $y^3 = 2x^2$ perpendicular to the line $x+2y-2=0$?

Your work:

① $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{\frac{4-(x+h)}{3+(x+h)} - \frac{4-x}{3+x}}{h}$

$= \lim_{h \to 0} \frac{\frac{4-x-h}{3+x+h} - \frac{4-x}{3+x}}{h} = \lim_{h \to 0} \left[\frac{(4-x-h)(3+x)-(4-x)(3+x+h)}{(3+x+h)(3+x)} \cdot \frac{1}{h} \right]$

$= \lim_{h \to 0} \frac{12+4x-3x-x^2-3h-hx-12-4x-4h+3x+x^2+xh}{(3+x+h)(3+x)\cdot h}$

$= \lim_{h \to 0} \frac{-7h}{(3+x+h)(3+x)\cdot h} = \lim_{h \to 0} \frac{-7}{(3+x+h)(3+x)} = \frac{-7}{(3+x)^2}$

(Confirm/Check with Quotient Rule.)

② $y^3 = 2x^2 \Rightarrow 3y^2 \cdot \frac{dy}{dx} = 4x \Rightarrow \frac{dy}{dx} = \frac{4x}{3y^2} \leftarrow (y \neq 0)$

$\perp$ $2y = -x+2$

$y = -\frac{x+2}{2} \Rightarrow m = -1/2$ so then $\frac{dy}{dx} = 2$

$\frac{4x}{3y^2} = 2 \Rightarrow 2x = 3y^2 \Rightarrow x^2 = \frac{9y^4}{4}$

$\Rightarrow y^3 = 2x^2 = 2 \cdot \left(\frac{9y^4}{4}\right) \Leftrightarrow y^3 = \frac{9y^4}{2} \Leftrightarrow 2y^3 - 9y^4 = 0$

$\Leftrightarrow y^3(2-9y) = 0 \Leftrightarrow y=0$ or $y = \frac{2}{9} \Rightarrow x = \frac{3y^2}{2} = \frac{3}{2} \cdot \frac{4}{81}$

impossible.

$x = \frac{2}{27}$; $y = \frac{2}{9}$ Point $\left(\frac{2}{27}, \frac{2}{9}\right)$

Name KEY **No Calculators. Present neatly. Score** ______. **G (10 min)**

1.Use the limit definition to find the derivative of f(x).

$$f(x)=\frac{3-x}{4+x}$$

2. Find the values of a and b for the curve $x^2y+ay^2=b$ if the point (1, 1) is on the graph and the tangent line at (1, 1) has the equation $4x+3y=7$.

Your work:

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$$2xy+x^2\cdot\frac{dy}{dx}+2ay\frac{dy}{dx}=0 \qquad \left.\frac{dy}{dx}\right|_{(1,1)}=\frac{-4}{3}$$

$$\Rightarrow \quad 2\cdot1\cdot1+1\cdot\left(\frac{-4}{3}\right)+2a\cdot1\cdot\left(\frac{-4}{3}\right)=0$$

$$2-\frac{4}{3}-\frac{8a}{3}=0$$

$$\frac{2}{3}=\frac{8}{3}a \Rightarrow \boxed{a=1/4}$$

Plug in (1, 1) into original eqtn: $1^2\cdot1+a\cdot1^2=b$

$$1+a=b$$

$$1+1/4=b$$

$$\boxed{\frac{5}{4}=b}$$

1

$$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to0}\left(\frac{3-(x+h)}{4+(x+h)}-\frac{3-x}{4+x}\right)\frac{1}{h}$$

$$=\lim_{h\to0}\frac{(3-x-h)(4+x)-(3-x)(4+x+h)}{h(4+x)(4+x+h)}$$

$$=\lim_{h\to0}\frac{\cancel{12}+\cancel{3x}-\cancel{4x}-\cancel{x^2}-4h-\cancel{hx}-\cancel{12}-3h+\cancel{x^2}+\cancel{xh}-\cancel{3x}+\cancel{4x}}{h(4+x)(4+x+h)}$$

$$=\lim_{h\to0}\frac{-7h}{h(4+x)(4+x+h)}=\frac{-7}{(4+x)^2}$$