Name SHUBLEKA **No calculators. Present neatly. Score**_____. **B x2**

1) /KEY

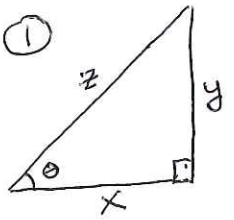
The hypotenuse of a right triangle is growing at a constant rate of a centimeters per second and one leg is decreasing at a constant rate of b centimeters per second. How fast is the acute angle between the hypotenuse and the other leg changing at the instant when both legs are 1 cm?

2) Find the limit or explain why it does not exist.

$$\lim_{x\to 1}\sqrt{\frac{\ln x}{x^4-1}}$$

Your work:

(1)



$\frac{dz}{dt} = a$ cm/s

$\frac{dy}{dt} = -b$ cm/s

$\frac{d\theta}{dt}\Big|_{x=1,\,y=1} = ?$

$\theta = \pi/4$ when $x=y=1$; $z=\sqrt{2}$

$\tan\theta = \frac{y}{x}$

or

$\sin\theta = \frac{y}{z}$

$\cos\theta \frac{d\theta}{dt} = \frac{z\cdot\frac{dy}{dt} - \frac{dz}{dt}\cdot y}{z^2}$ ←

$\Leftrightarrow \frac{d\theta}{dt} = \frac{1}{\cos\theta}\cdot\frac{z\cdot\frac{dy}{dt} - \frac{dz}{dt}\cdot y}{z^2}\Big|_{\theta=\pi/4,\,x=1,\,y=1,\,z=\sqrt{2}} = \sqrt{2}\cdot\frac{\sqrt{2}\cdot(-b) - a\cdot 1}{2} = -\left(b+\frac{\sqrt{2}}{2}a\right)$

When $x=1=y$, the acute angle is changing at a rate of $-\left(b+\frac{\sqrt{2}}{2}a\right)$

(2) $\lim_{x\to 1}\sqrt{\frac{\ln x}{x^4-1}} = \sqrt{\lim_{x\to 1}\frac{\ln x}{(x^4-1)}}$ (numerator $\to 0$, denominator $\to 0$) $= \sqrt{\lim_{x\to 1}\frac{1/x}{4x^3}} = \sqrt{\lim_{x\to 1}\frac{1}{4x^4}}$

$= \sqrt{\frac{1}{4}} = \boxed{\frac{1}{2}}$

Name SHUBLEKA **No calculators. Present neatly. Score_____. C x2**

1) KEY.

A particle is moving along the curve $y = x \ln x$. Find all values of x at which the rate of change of y with respect to time is three times that of x. [Assume that dx/dt is never zero.]

2) Find the limit or explain why it does not exist.

$\lim_{x \to 0} \frac{a^x - 1}{x}, \quad a > 0$

Your work:

(1) $y = x \cdot \ln x$ $\quad \frac{dy}{dt} = \frac{dx}{dt} \cdot \ln x + x \cdot \frac{1}{x} \cdot \frac{dx}{dt}$

$\left(\frac{dy}{dt} = \frac{dx}{dt}(\ln x + 1) \right) \Rightarrow \quad \frac{dy}{dt} = 3 \cdot \frac{dx}{dt}$

$3\frac{dx}{dt} = \frac{dx}{dt}(\ln x + 1)$ $\quad$ since $\frac{dx}{dt} \neq 0$

$\ln x + 1 = 3$

$\ln x = 2$

$\boxed{x = e^2}$

(2) $\lim_{x \to 0} \frac{a^x - 1}{x}$ ($a^x - 1 \to 0$, $x \to 0$) $= \lim_{x \to 0} \frac{\ln a \; a^x}{1} = \boxed{\ln a}$ ✓