Course: AP Calculus BC | Quiz: 10 | Instructor: D. Shubleka

Name SHUBLEKA /KEY. **No calculators. Present neatly. Score________.**

1)

Let $r(x) = f(g(h(x)))$, where $h(1) = 2$, $g(2) = 3$, $h'(1) = 4$, $g'(2) = 5$, and $f'(3) = 6$. Find $r'(1)$.

2)

If $F(x) = f(xf(xf(x)))$, where $f(1) = 2, f(2) = 3, f'(1) = 4$, $f'(2) = 5$, and $f'(3) = 6$, find $F'(1)$.

3)

If $xy + y^3 = 1$, find the value of y'' at the point where $x = 0$.

4)

Find the points on the curve $y = (\cos x)/(2 + \sin x)$ at which the tangent is horizontal.

Your work:

1) $r(x) = f\left(g[h(x)]\right)$

$r'(x) = f'\left(g[h(x)]\right) \cdot g'[h(x)] \cdot h'(x) \Rightarrow r'(1) = f'(g(h(1))) \cdot g'(h(1)) \cdot h'(1) =$

$r'(1) = f'(g(2)) \cdot g'(2) \cdot 4 = f'(3) \cdot 5 \cdot 4 = 6 \cdot 20 = 120$

2) $F(x) = f\left(x \cdot f[x \cdot f(x)]\right)$

$F'(x) = f'\left(x \cdot f[x \cdot f(x)]\right) \cdot \left[1\, f[x \cdot f(x)] + x \cdot f'[x\, f(x)]\,(f(x) + x f'(x))\right]$

$F'(1) = f'(f(f(1)))\ \left[f[f(1)] + f'(f(1)) \cdot (f(1) + f'(1))\right]$

$= f'(f(2))\ \left[f(2) + f'(2) \cdot (2 + 4)\right]$

$= f'(3)\ [3 + 5 \cdot 6] = 6 \cdot 33 = 198$

3) $xy + y^3 = 1$ @ $x = 0,\ y = 1$

$y + xy' + 3y^2 y' = 0 \Rightarrow$ @ $(0,1)$: $1 = -3y' \Rightarrow y'_{(0,1)} = -1/3$

$y' + xy'' + y' + 6yy' \cdot y' + 3y^2 y'' = 0 \Rightarrow$ @ $(0,1) \Rightarrow \frac{-1}{3} + \left(\frac{-1}{3}\right) + 6 \cdot \frac{1}{9} + 3y'' = 0$

$\frac{-2}{3} + \frac{2}{3} + 3y'' = 0 \Rightarrow y'' = 0.$

4) $y = \dfrac{\cos x}{2 + \sin x}$ $\quad \dfrac{dy}{dx} = \dfrac{(2 + \sin x)(-\sin x) + \cos x(-\cos x)}{(2 + \sin x)^2} = \dfrac{-2\sin x - \sin^2 x - \cos^2 x}{(2 + \sin x)^2}$

$\dfrac{dy}{dx} = \dfrac{-2\sin x - 1}{(2 + \sin x)^2} = 0 \Rightarrow \sin x = \dfrac{-1}{2}$ $\quad x = \dfrac{-\pi}{6} + 2k\pi$

$x = 7\pi/6 + 2k\pi$

or $x = -5\pi/6 + 2k\pi$

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1) / KEY.

If g is a twice differentiable function and $f(x) = xg(x^2)$, find f'' in terms of g, g', and g''.

2)

If $F(x) = f(3f(4f(x)))$, where $f(0) = 0$ and $f'(0) = 2$, find $F'(0)$.

3)

If $x^2 + xy + y^3 = 1$, find the value of y''' at the point where $x = 1$.

4)

For what values of x does the graph of $f(x) = x + 2\sin x$ have a horizontal tangent?

Your work:

1) $f(x) = x \cdot g(x^2)$

$f'(x) = 1 \cdot g(x^2) + x \cdot g'(x^2) \cdot 2x = g(x^2) + 2x^2 g'(x^2)$

$f''(x) = g'(x^2) \cdot 2x + 2x^2 \cdot g''(x^2) \cdot 2x + 4x \cdot g'(x^2)$

$f''(x) = 6x\, g'(x^2) + 4x^3 g''(x^2)$

2) $F(x) = f(3 \cdot f[4 f(x)])$

$F'(x) = f'(3 f[4 f(x)]) \cdot 3 f'[4 \cdot f(x)] \cdot 4 \cdot f'(x)$

$F'(0) = f'(3 f[4 \cdot f(0)]) \cdot 3 f'[4 \cdot f(0)] \cdot 4 f'(0)$

$F'(0) = f'(0) \cdot 12 \cdot f'(0) \cdot f'(0) = 2 \cdot 12 \cdot 2 \cdot 2 = 96.$

3) $x^2 + xy + y^3 = 1 \quad x = 1 \Rightarrow y = 0 \quad (1, 0)$

$2x + x \cdot y' + y + 3y^2 \cdot y' = 0 \Longrightarrow @(1,0) : 2 + y' + 0 + 0 = 0 \Rightarrow y'_{(1,0)} = -2.$

$2 + xy'' + y' + y' + 6y y' y' + 3y^2 y'' = 0 \Rightarrow @(1,0) : 2 + y'' - 2 - 2 + 0 = 0$

$xy'' + 2y' + 6y[y']^2 + 3y^2 y'' = 0$

$y'' + xy''' + 2y'' + 6y'(y')^2 + 12 y y' y''$

$3y^2 y''' + 6y^2 y' y'' = 0$

$y''(1) = 2. \qquad \Rightarrow y''_{(1,0)} = +4 - 2 = 2.$

4) $f(x) = x + 2\sin x \Rightarrow f'(x) = 1 + 2\cos x = 0 \Rightarrow \cos x = -1/2$

$x = \frac{2\pi}{3} + 2k\pi$

$x = \frac{-2\pi}{3} + 2k\pi$

or

$x = \frac{4\pi}{3} + 2k\pi$

$y'_{(1,0)} = -2$

$y''_{(1,0)} = 2 \quad$ so.... $2 + y''' + 4 + 6 \cdot (-2)^3 = 0$

$x = 1, y = 0 \quad y''' = 48 - 6 = 42$