Name SHUBLEKA /KEY **No Calculators. Present neatly. Score________.**

1)Find an equation of the tangent line to the curve at the given point.

$$y = x + \tan x, \quad (\pi, \pi)$$

2)

If $H(\theta) = \theta \sin \theta$, find $H'(\theta)$ and $H''(\theta)$.

3)

For what values of x does the graph of $f(x) = x + 2 \sin x$ have a horizontal tangent?

4) Find the limit or explain why it does not exist.

$$\lim_{x \to \pi/4} \frac{1 - \tan x}{\sin x - \cos x}$$

Your work:

1) $y = x + \tan x$ @ (π, π)

$\frac{dy}{dx} = 1 + \sec^2 x$ $\quad \left.\frac{dy}{dx}\right|_{x=\pi} = 1 + \sec^2 \pi = 1 + (-1)^2 = 2$ $\quad$ T: $y - \pi = 2(x - \pi)$

$y = 2x - \pi$

2) $H(\theta) = \theta \cdot \sin\theta$ $\quad H'(\theta) = 1 \cdot \sin\theta + \theta \cos\theta = \sin\theta + \theta\cos\theta$

$H''(\theta) = \cos\theta + 1 \cdot \cos\theta + \theta \cdot (-\sin\theta)$

$H''(\theta) = 2\cos\theta - \theta \sin\theta$

3) $f(x) = x + 2\sin x$ $\quad f'(x) = 1 + 2\cos x = 0 \Rightarrow \cos x = -\frac{1}{2}$ $\quad x = \frac{2\pi}{3} + 2k\pi$

The tangent line on $f(x)$ is horizontal when $x = \pm\frac{2\pi}{3} + 2k\pi$. $\quad x = \frac{4\pi}{3} + 2k\pi$

4) $\lim_{x \to \pi/4} \frac{(1 - \tan x) \to 0}{(\sin x - \cos x) \to 0} = \lim_{x \to \pi/4} \frac{\left(1 - \frac{\sin x}{\cos x}\right)}{\sin x - \cos x} = \lim_{x \to \pi/4} \frac{(\cos x - \sin x)}{\cos x(\sin x - \cos x)} =$

$= \lim_{x \to \pi/4} \frac{-(\sin x - \cos x)}{\cos x(\sin x - \cos x)} = \lim_{x \to \pi/4} \frac{-1}{\cos x} = \frac{-1}{\frac{\sqrt{2}}{2}} = -\sqrt{2}$

Name SHUBLEKA **No Calculators. Present neatly. Score________.**

1) Find an equation of the tangent line to the curve at the given point.

/ KEY.

$$y = \sec x, \quad (\pi/3, 2)$$

2)

If $f(t) = \csc t$, find $f''(\pi/6)$.

3)

Find the points on the curve $y = (\cos x)/(2 + \sin x)$ at which the tangent is horizontal.

4) Find the limit or explain why it does not exist.

$$\lim_{x \to 1} \frac{\sin(x-1)}{x^2 + x - 2}$$

Your work:

1) $f(x) = \sec x \quad @ \; (\pi/3, 2)$

$f'(x) = \sec x \tan x \qquad f'(x)\Big|_{\pi/3 = x} = \sec\frac{\pi}{3} \tan\frac{\pi}{3} = 2 \cdot \sqrt{3} = 2\sqrt{3}.$

Tangent: $y - 2 = 2\sqrt{3}\,(x - \pi/3)$ or $y = 2 + 2\sqrt{3}\,x - \frac{2\sqrt{3}\,\pi}{3}$

or $y = 2 - \frac{2\sqrt{3}\,\pi}{3} + 2\sqrt{3}\,x$

2) $f(t) = \csc t$

$f''(t) = ?$

$f'(t) = -\csc t \cot t$

$f''(t) = -(\csc t \cot t) \cdot \cot t + (-\csc t) \cdot (-\csc^2 t)$

$f''(t) = \csc t \cot^2 t + \csc^3 t = \csc t \left(\cot^2 t + \csc^2 t\right)$

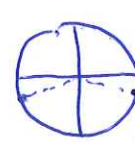
$f''(\pi/6) = 2 \left[\, 3 + 4 \,\right] = 14.$

3) $y = \dfrac{\cos x}{2 + \sin x}$

$$\frac{dy}{dx} = \frac{(2+\sin x)(-\sin x) - \cos x \cdot \cos x}{(2+\sin x)^2} = \frac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2}$$

$$= \frac{-2\sin x - 1}{(2+\sin x)^2} = 0 \implies \sin x = -1/2$$

$$\begin{cases} x = -\frac{\pi}{6} + 2k\pi \\ x = -\frac{5\pi}{6} + 2k\pi. \end{cases}$$



4) $\displaystyle \lim_{x \to 1} \frac{\sin(x-1) \to 0}{x^2 + x - 2 \to 0} = \lim_{x \to 1} \frac{\sin(x-1)}{(x-1)(x+2)} =$

$$= \lim_{x \to 1} \left(\frac{\sin(x-1)}{x-1} \cdot \frac{1}{x+2} \right) = \lim_{u \to 0} \frac{\sin u}{u} \cdot \lim_{x \to 1} \frac{1}{x+2} = 1 \cdot \frac{1}{3} = \frac{1}{3}.$$

$u = x - 1$