Problem 1

$$f(x)=\sqrt{9-x}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=$$

$$=\lim_{h\to 0}\frac{\sqrt{9-(x+h)}-\sqrt{9-x}}{h}$$

$$=\lim_{h\to 0}\frac{\left(\sqrt{9-(x+h)}-\sqrt{9-x}\right)\left(\sqrt{9-(x+h)}+\sqrt{9-x}\right)}{h\left(\sqrt{9-(x+h)}+\sqrt{9-x}\right)}$$

$$=\lim_{h\to 0}\frac{\left(9-(x+h)-(9-x)\right)}{h\left(\sqrt{9-(x+h)}+\sqrt{9-x}\right)}$$

$$=\lim_{h\to 0}\frac{-h}{h\left(\sqrt{9-(x+h)}+\sqrt{9-x}\right)}$$

$$=\lim_{h\to 0}\frac{-1}{\left(\sqrt{9-(x+h)}+\sqrt{9-x}\right)}=\frac{-1}{2\sqrt{9-x}}$$

Problem 2

$$f(x)=\frac{x^2-1}{2x-3}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=$$

$$=\lim_{h\to 0}\frac{\frac{(x+h)^2-1}{2(x+h)-3}-\frac{x^2-1}{2x-3}}{h}$$

$$=\lim_{h\to 0}\frac{\left((x+h)^2-1\right)(2x-3)-\left(x^2-1\right)\left(2(x+h)-3\right)}{h\left(2(x+h)-3\right)(2x-3)}$$

$$=\lim_{h\to 0}\frac{\left(x^2+2xh+h^2-1\right)(2x-3)-\left(x^2-1\right)(2x+2h-3)}{h\left(2(x+h)-3\right)(2x-3)}$$

$$=\lim_{h\to 0}\frac{2x^3-3x^2+4x^2h-6xh+2xh^2-3h^2-2x+3-\left[2x^3+2x^2h-3x^2-2x-2h+3\right]}{h\left(2(x+h)-3\right)(2x-3)}$$

$$=\lim_{h\to 0}\frac{4x^2h-6xh+2xh^2-3h^2-2x^2h+2h}{h\left(2(x+h)-3\right)(2x-3)}$$

$$=\lim_{h\to 0}\frac{2x^2h-6xh+2xh^2-3h^2+2h}{h\left(2(x+h)-3\right)(2x-3)}=\lim_{h\to 0}\frac{2x^2-6x+2xh-3h+2}{\left(2(x+h)-3\right)(2x-3)}=\frac{2(x^2-3x+1)}{(2x-3)^2}$$