Present neatly on separate paper. Justify for full credit. No Calculators.
Name ______________________ Score ______ ~15 minutes

1. True or False? Explain. [5 points]

a)

If $|f|$ is continuous at a, so is f.

False. Consider: $f(x)=\begin{cases}1 & x\ge 0\\ -1 & x<0\end{cases}$. Sketch $|f(x)|$ to verify that it is continuous.

b)

The equation $x^{10}-10x^2+5=0$ has a root in the interval $(0, 2)$.

True. Observe that the right hand side is a polynomial (continuous everywhere). If we apply IVT on [0, 2], it does not guarantee the existence of roots. However, if we apply IVT on the intervals [0, 1] and/or [1, 2], we get a sign change. Therefore, by IVT, the given equation has a root (at least two!) in the interval (0, 2).

c)

If $f(1)>0$ and $f(3)<0$, then there exists a number c between 1 and 3 such that $f(c)=0$.

False. If f(x) is continuous, then the given statement would be true. Continuity is a requirement for the conclusion of the IVT.

d)

If $\lim_{x\to 6}[f(x)\,g(x)]$ exists, then the limit must be $f(6)\,g(6)$.

False. The function f(x) g(x) need not be continuous in order for the limit to exist.

e)

If neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists, then $\lim_{x\to a}[f(x)+g(x)]$ does not exist.

False. Consider: $f(x)=\begin{cases}1 & x>0\\ -1 & x<0\end{cases}$ $g(x)=\begin{cases}-1 & x>0\\ 1 & x<0\end{cases}$.

2. Find the limit or explain why it does not exist. [5 points]

a)

$$\lim_{x\to 0}\frac{1-\sqrt{1-x^2}}{x}=\lim_{x\to 0}\frac{\left(1-\sqrt{1-x^2}\right)\left(1+\sqrt{1-x^2}\right)}{x\left(1+\sqrt{1-x^2}\right)}$$

$$=\lim_{x\to 0}\frac{1-\left(1-x^2\right)}{x\left(1+\sqrt{1-x^2}\right)}=\lim_{x\to 0}\frac{\left(x^2\right)}{x\left(1+\sqrt{1-x^2}\right)}=\lim_{x\to 0}\frac{x}{\left(1+\sqrt{1-x^2}\right)}=\frac{0}{2}=0$$

b)

$$\lim_{x\to 1}\left(\frac{1}{x-1}+\frac{1}{x^2-3x+2}\right)$$

$$\lim_{x\to 1}\frac{1}{x-1}+\frac{1}{x^2-3x+2}=$$

$$=\lim_{x\to 1}\frac{1}{x-1}+\frac{1}{(x-1)(x-2)}$$

$$=\lim_{x\to 1}\frac{x-2}{\left(x-1\right)(x-2)}+\frac{1}{(x-1)(x-2)}$$

$$=\lim_{x\to 1}\frac{x-1}{\left(x-1\right)(x-2)}=\lim_{x\to 1}\frac{1}{(x-2)}=-1$$