Present neatly on separate paper. Justify for full credit. No Calculators.

Name _____ Score _____ ~10 minutes

1.

Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \leqslant x < 3 \\ 2x - a + b & \text{if } x \geqslant 3 \end{cases}$$

[5 points]

2.

Briefly explain the Intermediate Value Theorem to a non-mathematical person. Why is it important? [5 points]

3. Find the limit or explain why it does not exist. [5 points]

a)

$$\lim_{h \to 0} \frac{\sin h}{1 - \cos h}$$

b)

$$\lim_{x \to 0} \frac{\sin 6x}{\sin 8x}$$

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1) Set the one-sided limits equal to each other and to the y-value:

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{(x-2)(x+2)}{(x-2)} = 4$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} ax^2 - bx + 3 = 4a - 2b + 3$$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} ax^2 - bx + 3 = 9a - 3b + 3$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} 2x - a + b = 6 - a + b$$

$$\begin{cases} 4a - 2b + 3 = 4 \\ 6 - a + b = 9a - 3b + 3 \end{cases} \rightarrow \begin{cases} 4a - 2b = 1 \\ 10a - 4b = 3 \end{cases} \rightarrow \begin{cases} -8a + 4b = -2 \\ 10a - 4b = 3 \end{cases}$$

$$\rightarrow \begin{cases} 2a = 1 \\ 10a - 4b = 3 \end{cases} \rightarrow \begin{cases} a = \dfrac{1}{2} \\ b = \dfrac{1}{2} \end{cases}$$

2) See Textbook or Class Notes on IVT 3) a) DNE b) 3/4