Present neatly. Justify for full credit. No Calculators.

Name ______________________ Score ______ ~10 minutes

1. Find the limit:

$$\lim_{x\to 3}\frac{\sqrt{x+6}-x}{x^3-3x^2}$$

2.
Show that the equation has a solution in the given interval.

$$x^5-x^3+3x-5=0,\quad (1,2)$$

1.

$$\lim_{x\to 3}\frac{\sqrt{x+6}-x}{x^3-3x^2}=$$

$$=\lim_{x\to 3}\frac{\left(\sqrt{x+6}-x\right)\left(\sqrt{x+6}+x\right)}{x^2\left(x-3\right)\left(\sqrt{x+6}+x\right)}=$$

$$=\lim_{x\to 3}\frac{x+6-x^2}{x^2\left(x-3\right)\left(\sqrt{x+6}+x\right)}=$$

$$=\lim_{x\to 3}\frac{-(x-3)(x+2)}{x^2\left(x-3\right)\left(\sqrt{x+6}+x\right)}=$$

$$=\lim_{x\to 3}\frac{-(x+2)}{x^2\left(\sqrt{x+6}+x\right)}=\frac{-5}{9*6}=\frac{-5}{54}$$

2. The left hand side of the equation is a polynomial function. All polynomial functions are continuous everywhere in their domain (all real numbers). In particular, this polynomial is continuous on the closed interval [1, 2]. Therefore we can try to apply the Intermediate Value Theorem on this interval:

$$f(x)=x^5-x^3+3x-5$$
$$f(1)=1-1+3-5=-2<0$$
$$f(2)=32-8+6-5=25>0$$

In order to move in a continuous fashion from (1, -2) to (2, 25), the graph must intersect the horizontal axis at least once. Therefore, by the Intermediate Value Theorem, the equation must have at least one solution on the open interval (1, 2).