AP Calculus BC Mr. Shubleka Quiz: 51

Present neatly on separate paper. Justify for full credit. No Calculators.

Name KEY/SHUBLEKA Score ______ A (10 minutes) x1

1)

$$\sum_{n=0}^{\infty}\frac{(-1)^n(\pi)^{2n}}{(2n)!} = \cos \pi = -1.$$

2)

$a_n = (-1)^{n+1} \cdot \frac{x^n}{n}$

What are the values of x for which the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$ converges?

$\left| \frac{x^{n+1}}{n+1} \cdot \frac{n}{x^n} \right| = |x| \cdot \frac{n}{n+1} \to |x| < 1$ $\quad -1 < x < 1$

$-1 < x \leq 1$ $\quad \begin{cases} x = 1 & \text{Alt Harmonic} \to \text{converges} \\ x = -1 & \text{Harmonic} \to \text{diverges} \end{cases}$

3)

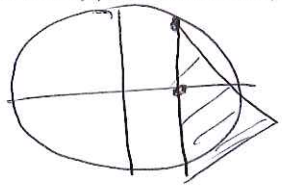
The first three nonzero terms in the Maclaurin series about $x = 0$ of xe^{-x} are

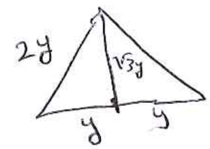
$x \cdot e^{-x} = x\left[1 + (-x) + \frac{(-x)^2}{2!} + \ldots + \frac{(-x)^n}{n!} + \ldots\right]$

$= x - x^2 + \frac{x^3}{2!} + \ldots$

4)

A solid has a circular base of radius 3. If every plane cross section perpendicular to the x-axis is an equilateral triangle, then its volume is ________.





$A = \frac{2y \cdot \sqrt{3}y}{2} = \sqrt{3}\,y^2$

$A = \sqrt{3}\,(9 - x^2)$

$V = \sqrt{3}\int_{-3}^{3} 9 - x^2 \, dx$

$= 2\sqrt{3}\left(9x - \frac{x^3}{3}\right)\Big|_0^3$

$= 2\sqrt{3} \cdot 18$

$= \underline{36\sqrt{3}}$

5)

$$\int_{-1}^{1}\frac{dx}{x^2+5x+6} = \int_{-1}^{1}\frac{1}{(x+2)(x+3)}\,dx = \ln|x+2| - \ln|x+3|$$

$$= \ln\left|\frac{x+2}{x+3}\right| \Big|_{-1}^{1}$$

$\frac{A}{x+2} + \frac{B}{x+3} = \frac{1}{x+2} - \frac{1}{x+3}$

$= \ln \frac{3}{4} - \ln \frac{1}{2}$

$= \ln\left(\frac{3/4}{1/2}\right) = \ln \frac{3}{2}$

Present neatly on separate paper. Justify for full credit. No Calculators.

Name KEY/SHUBLEKA Score ______ F (10 minutes) **x1**

1)

. What is the sum of the series $\frac{3}{2}-\frac{3}{8}+\frac{3}{32}-\frac{3}{128}+\ldots$?

Geometric Series: a=3/2 r=-1/4. It converges to a / (1-r) = 6/5

2)

What are all the values of x for which $\sum_{n=2}^{\infty}\frac{(-1)^n}{\ln n}x^n$ converges?

$\boxed{-1 < x \le 1}$

$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+1}}{\ln(n+1)}\cdot\frac{\ln n}{x^n}\right| = |x|\cdot\frac{\ln(n)}{\ln(n+1)} \longrightarrow |x|\cdot\frac{1/n}{1/(n+1)} \longrightarrow |x| < 1$

$-1 < x < 1$

$x=1$: Converges by Alt. Series Test.

$x=-1$: $\sum\frac{1}{\ln n}$ vs $\sum\frac{1}{n}$.

3)

The Maclaurin series for $\frac{\sin(x^2)}{x^2}$ is

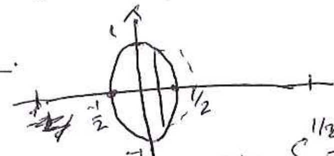
$u = x^2$

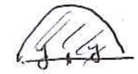
$\frac{\sin u}{u} = \frac{u - \frac{u^3}{3!} + \frac{u^5}{5!} - \ldots}{u} = 1 - \frac{u^2}{3!} + \frac{u^4}{5!} - \ldots$

$= 1 - \frac{x^4}{3!} + \frac{x^8}{5!} - \ldots = \sum_{n=0}^{\infty}\frac{(-1)^n \cdot x^{4n}}{(2n+1)!}$

4)

The base of a solid is the region enclosed by the ellipse $4x^2+y^2=1$. If all plane cross sections perpendicular to the x-axis are semicircles, then its volume is ________.





$A(x) = \pi\frac{r^2}{2} = \pi\frac{y^2}{2}$

$= \pi\cdot\frac{(1-4x^2)}{2}$

$V = \int_{-1/2}^{1/2}\pi\frac{1-4x^2}{2}dx = \pi\int_0^{1/2}1-4x^2\,dx$

$= \pi\cdot\left(x-\frac{4x^3}{3}\right)\Big|_0^{1/2}$

$= \pi\left(\frac{1}{2}-\frac{1}{6}\right)$

$= \frac{\pi}{3}$ c.u.

5)

$\int_0^2 \ln x\,dx = \lim_{b\to0^+}\int_b^2 \ln x\,dx$

$\int 1\cdot\ln x\,dx$ ($1 = v'$, $\ln x = u$)

$= uv - \int u'v\,dx$

$= x\cdot\ln x - \int\frac{1}{x}\cdot x\,dx$

$= x\cdot\ln x - x + C$

$= \lim_{b\to0^+}(x\cdot\ln x - x)\Big|_b^2$

$= \lim_{b\to0^+}(2\cdot\ln2-2)-(b\cdot\ln b - b)$

$= 2\ln2-2-\lim_{b\to0^+}b\cdot\ln b$

$= \lim_{b\to0^+}\frac{\ln b}{1/b} = \lim_{b\to0^+}\frac{1/b}{-1/b^2} = \lim_{b\to0^+}-b = 0.$

$= 2\ln2-2.$