

Present neatly on separate paper. Justify for full credit. No Calculators.

Name Key / D.S. Score _____ A (10 minutes) **x1**

1)

Use Euler's Method to approximate $f(1)$ with step size $\Delta x = 0.5$ if $y = f(x)$ is a solution and the initial condition is $(0, 1)$.

$$\frac{dy}{dx} = x + y + 1$$

(see other section)

Illustrate your solution with a graph.

2)

Solve the following differential equation. Find the specific solution by using the initial condition $(1, 3)$.

$$\frac{dy}{dt} \frac{1+t^2}{y} = 1$$

$$\frac{1}{y} dy = \frac{1}{1+t^2} dt$$

$$\int \frac{1}{y} dy = \int \frac{1}{1+t^2} dt$$

$$\ln|y| = \arctan t + C$$

$$|y| = e^{\arctan t + C}$$

$$y = \pm e^{\arctan t + C}$$

$$3 = e^{\arctan 1 + C}$$

$$3 = e^{\pi/4 + C} \rightarrow C = \ln 3 - \frac{\pi}{4}$$

$$y = e^{\arctan t} \cdot \frac{e^{\ln 3}}{e^{\pi/4}} = \frac{3 \cdot e^{\arctan t}}{e^{\pi/4}}$$

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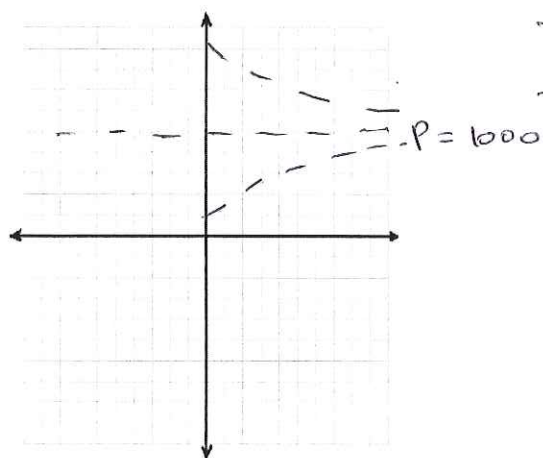
Name KEY/DS- Score _____ F (10 minutes) x1
1)

Describe the differential equation in as much detail as possible.

$$\frac{dP}{dt} = 0.1P\left(1 - \frac{P}{1000}\right)$$

$0 < P < 1000 \Rightarrow \frac{dP}{dt} > 0$ population grows.
 $1000 < P \Rightarrow \frac{dP}{dt} < 0$ population decreases.

What happens to the population in the long run if $P_0=750$? What if $P_0=1300$? Explain.



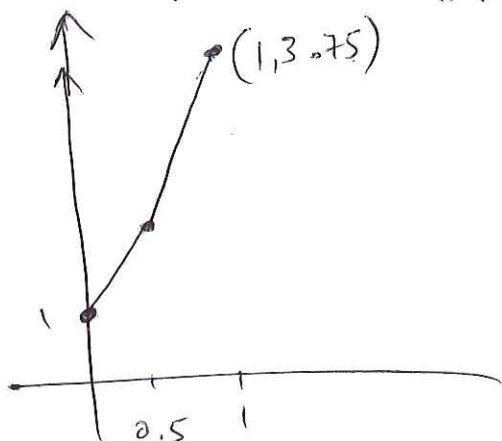
- If $P_0 = 750$, the population grows.
- If $P_0 = 1300$, the population decreases.
- In both situations, $P(t)$ approaches 1000.

2)

Use Euler's Method to approximate $f(1)$ with step size $\Delta x = 0.5$ if $y = f(x)$ is a solution and the initial condition is $(0, 1)$.

$$\frac{dy}{dx} = x + y + 1$$

Illustrate your solution with a graph.



x	y	dy/dx	
0	1	2	$y - 1 = 2(x - 0)$
0.5	2	3.5	$y - 2 = 3.5(x - 0.5)$
1	3.75		$y = 2 + \frac{3.5}{2}$

$f(1) \approx 3.75$