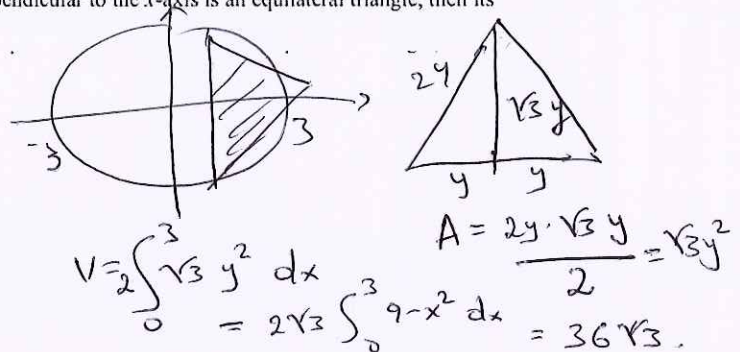Present neatly on separate paper. Justify for full credit. No Calculators.
Name ___KEY/SHUBLEKA___ Score _____ 10 minutes **Weight: x1**

1)

A solid has a circular base of radius 3. If every plane cross section perpendicular to the x-axis is an equilateral triangle, then its volume is _____.

$$V = 2 \int_0^3 \sqrt{3}\, y^2\, dx = 2\sqrt{3} \int_0^3 9 - x^2\, dx = 36\sqrt{3}.$$

$$A = \frac{2y \cdot \sqrt{3}\, y}{2} = \sqrt{3}\, y^2$$

2)

$$\frac{d}{dx} \int_x^0 \frac{du}{1+u^2} = \frac{-1}{1+x^2} \quad (FTC)$$

3)

At each point (x,y) on a certain curve, the slope of the curve is $4xy$. If the curve contains the point $(0,4)$, then its equation is _____.

$$\frac{dy}{dx} = 4xy \implies \int \frac{1}{y}\, dy = \int 4x\, dx \qquad \ln|y| = 2x^2 + C$$

$$y = \pm\, e^{2x^2 + C} \implies y = e^{2x^2 + C}$$

$$C = \ln 4$$

$$y = 4 \cdot e^{2x^2}.$$

4)

A particle with velocity at any time t given by $v(t) = 2e^{2t}$ moves in a straight line. How far does the particle travel during the time interval when its velocity increases from 2 to 4?

$$v = 2 \implies 2 = 2e^{2t} \quad t = 0$$
$$v = 4 \implies 4 = 2 \cdot e^{2t} \implies 2t = \ln 2$$
$$t = \frac{\ln 2}{2}$$

$$\int_0^{\ln 2/2} |v(t)|\, dt = \int_0^{\ln 2/2} 2e^{2t}\, dt = e^{2t}\Big|_0^{\frac{\ln 2}{2}}$$

$$= 2 - 1 = 1 \text{ unit}$$

5)

$$\int_1^3 \frac{x}{x^2+1}\, dx =$$

$$= \frac{1}{2} \int_1^3 \frac{2x}{x^2+1}\, dx$$

$$= \frac{1}{2} \ln|x^2+1| \Big|_1^3$$

$$= \frac{1}{2} \left(\ln 10 - \ln 2 \right)$$

$$= \frac{1}{2} \ln 5 = \frac{\ln 5}{2}.$$