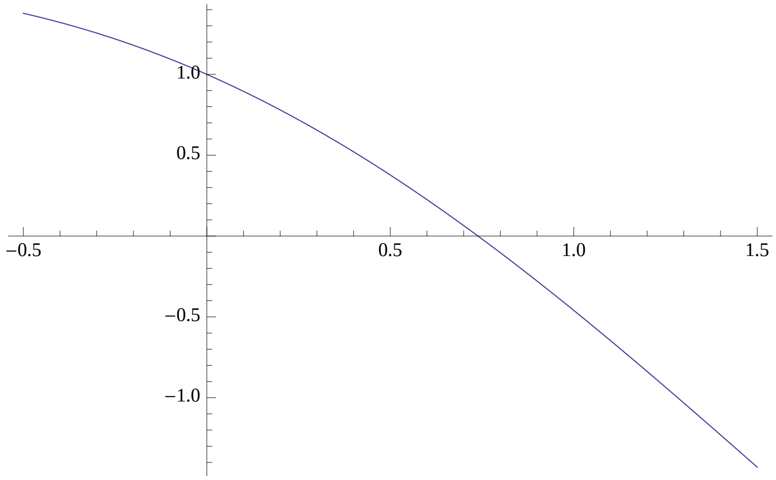
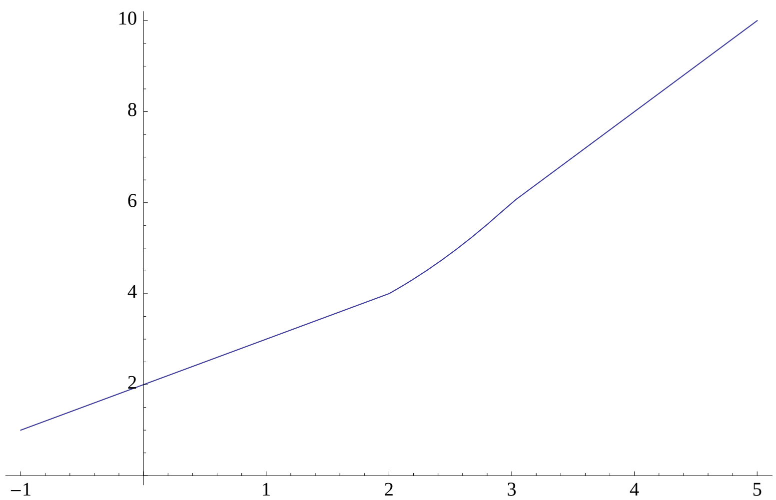
```
(* AP Calculus BC | Quiz 4 | Problem 1 *)
Plot[Cos[x] - x, {x, -0.5, 1.5}]
```



Rewrite the equation as $\cos(x) - x = 0$. Note that if $f(x) = \cos(x) - x$, then f(x) is continuous throughout its domain. In particular, it is continuous on the closed interval [0, 1]. We can hence apply the Intermediate Value Theorem. Observe that cos(0) - 0 = 1 > 0 and also that cos(1) - 1 < 1. Given that the sign changes, and the graph is continuous on the given interval, by the Intermediate Value Theorem, the function must have at least one zero on (0, 1). Therefore, the given equation must have at least one root in (0, 1).

```
(* AP Calculus BC | Quiz 4 | Problem 2 *)

Plot[Piecewise[{{(x^2 - 4) / (x - 2), x < 2},
   {0.5 x^2 - 0.5 x + 3, 2 < x < 3}, {2 x - 0.5 + 0.5, x > 3}}], {x, -1, 5}]
```



Set the one-sided limits equal to each other at x=2 and x=3. Solving the resulting system of equations with unknowns a and b, we get a = b = 0.5.

```
(* AP Calculus BC | Quiz 4 | Problem 1 *)
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Apply IVT to $f(x) = x^4 + x - 3$ on [1, 2]. Confirm that it is continuous on this interval. Write your conclusion as a full sentence, using the Intermediate Value Theorem.

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(* AP Calculus BC | Quiz 4 | Problem 2 *)
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The sentence translates into: $x^3 + 1 = x$.
Consider $f(x) = x^3 - x + 1 = 0$. This is a cubic function that starts in QIII and ends in QI. Apply the Intermediate Value Theorem on a closed interval such as [-2, 1]. Observe that f(-2) is negative and f(1) is positive. By IVT, the graph must cross the horizontal axis at least once. Equivalently, the given equation must have at least one real root; therefore such number does exist.