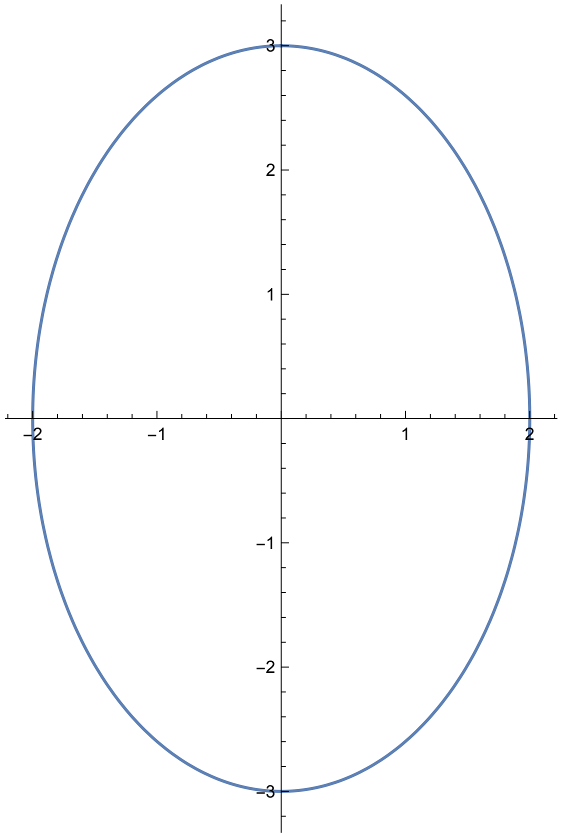
```
(* Problem 1 *)

ParametricPlot[{2 Sin[t], 3 Cos[t]}, {t, 0, 2 Pi}]
```




```
Simplify[D[3 Cos[t], t] / D[2 Sin[t], t]]
```

$$-\frac{3\,\text{Tan}[t]}{2}$$

```
Simplify[D[%, t] / D[2 Sin[t], t]]
```

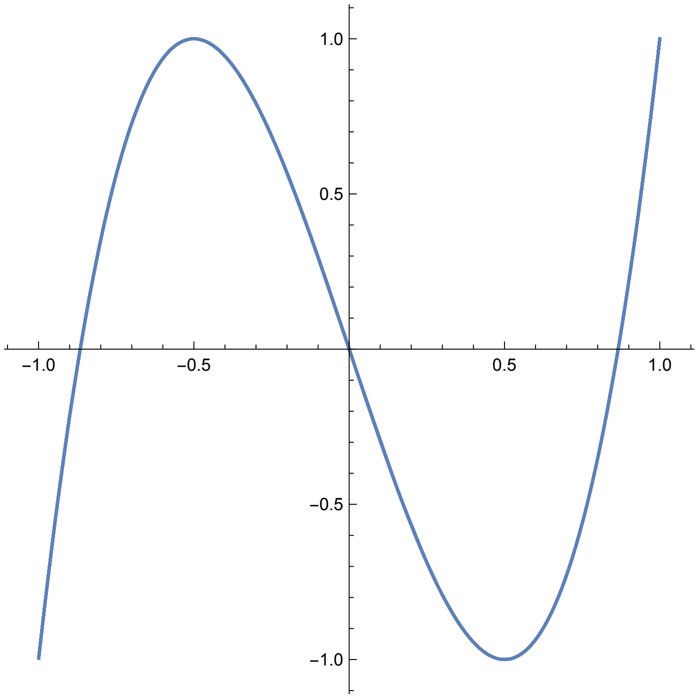
$$-\frac{3}{4}\,\text{Sec}[t]^3$$

The second derivative is positive whenever cos(t) is negative, and negative whenever cos(t) is positive. For the given domain, therefore, the graph is concave up on (pi / 2, 3pi/2) and concave down on (0, pi/2) and (3pi/2, 2pi).

Note that in the graph above, t=0 corresponds with the north pole, and the motion along the ellipse is clock-wise.

```
(* Problem 2 *)
```

```
ParametricPlot[{Cos[t], Cos[3 t]}, {t, -4 Pi, 4 Pi}]
```




```
D[Cos[t], t]
```

```
-Sin[t]
```

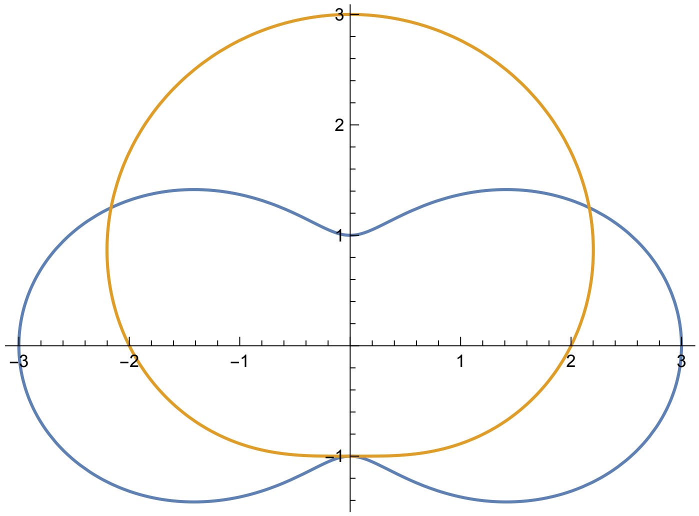
```
D[Cos[3 t], t]
```

```
-3 Sin[3 t]
```

A horizontal tangent occurs when dy/dt is zero but dx/dt is not zero. This happens when 3 t = 0, Pi, 2Pi, etc...
So t = 0, Pi/3, 2pi/3, k Pi/3, ... dx/dt is nonzero at t= Pi/3, 2Pi/3, ... so there are two horizontal tangents, at (-1/2, 1) and (1/2, -1).
A vertical tangent occurs when dx/dt is zero but dy/dt is not zero. This never happens. At the points t=0, pi, 2pi etc. both dy/dt and dx/dt are zero An investigation with L'Hospital's Rule concludes that there is no horizontal or vertical tangent line at these points.

```
(* Problem 3 *)
```

```
PolarPlot[{2 + Cos[2 t], 2 + Sin[t]}, {t, 0, 2 Pi}]
```



If we set the curves' equations equal to each other, we find solutions at t = Pi/6 and t= -Pi/2. We first set up an integral for the area in quadrants 1 and 4, and then double it to find total area. The precise area is 51 Sqrt[3] / 16.

```
2 * Integrate[0.5 (2 + Cos[2 t]) ^2 - 0.5 * (2 + Sin[t]) ^2, {t, -Pi / 2, Pi / 6}]
```

```
5.52091
```

```
(* Problem 4 *)
```

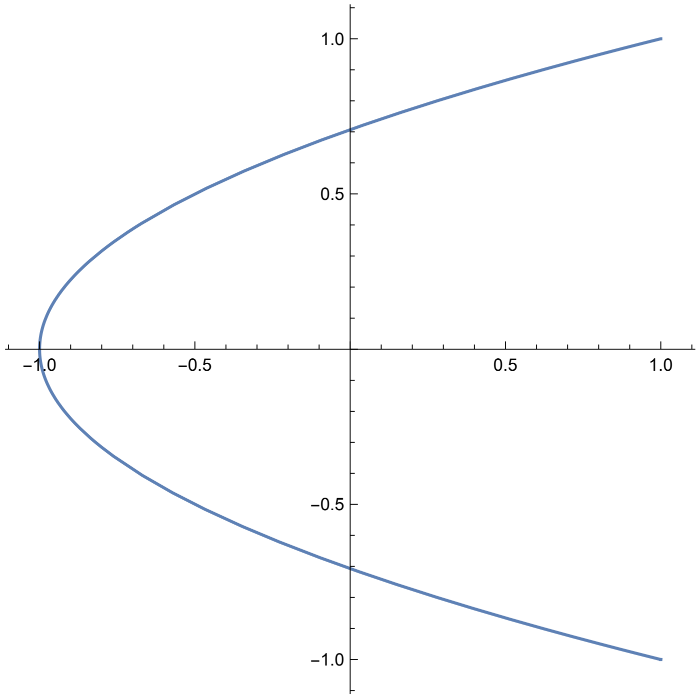
Consider f(x) = x sin(1/x). Rewrite it as f(x) = (sin(1/x)) / (1/x), and then use L'Hospital Rule to determine the limit of the squence. What does the Divergence Test say about this series?

```
(* Problem 5*)
```

f(x) = 10^x. You can either compute the first few derivatives, or rewrite the function as f(x) = e^(ln 10 * x). Substitute u = ln(10) x into the Maclaurin Series for e^u. The radius of convergence is infinity.

```
(* Problem 1*)
```

```
ParametricPlot[{Cos[2 t], Cos[t]}, {t, 0, Pi}]
```



The parametric curve will be concave up when the second derivative is positive, and concave down when negative. Below we compute the second derivative.

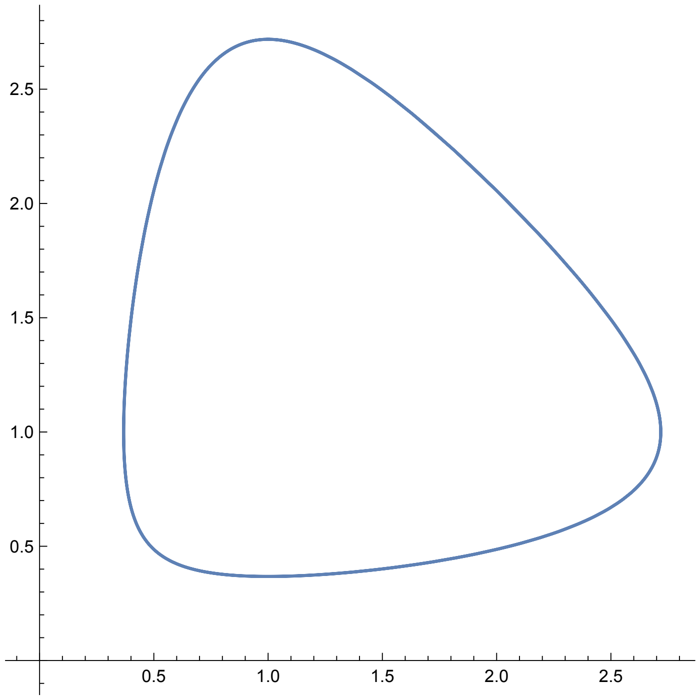
```
Simplify[D[D[Cos[t], t] / D[Cos[2 t], t], t] / D[Cos[2 t], t]]
```

$$-\frac{1}{16} \text{Sec}[t]^3$$

This expression is positive when cos(t) is negative. This occurs on the interval (Pi/2, Pi). Therefore, the curve is concave up on (Pi/2, Pi).
This expression is negative when cos(t) is positive. This occurs on the interval on (0, Pi/2). Therefore, the curve is concave down on (0, Pi/2).
Note that the particle starts traveling at the point (1, 1) on the parabola, which corresponds with t=0.

```
(* Problem 2*)
```

```
ParametricPlot[{E^(Sin[t]), E^(Cos[t])}, {t, -2 Pi, 2 Pi}]
```



The horizontal tangent will occur whenever dy/dt is zero but dx/dt is not zero. The vertical tangent will occur whenever dx/dt is zero but dy/dt is nonzero.

```
D[E^Sin[t], t]
```

$e^{\text{Sin}[t]}$ Cos[t]

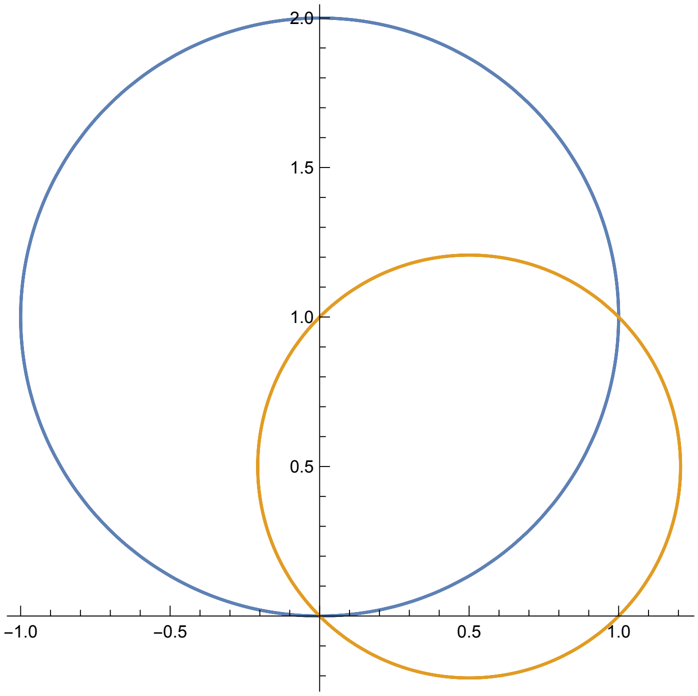
```
D[E^Cos[t], t]
```

$-e^{\text{Cos}[t]}$ Sin[t]

Vertical tangents occur at t = Pi/2, 3Pi2, ... etc. (add k Pi for general solution)
Horizontal tangents occur at t = 0, Pi, 2pi, ... etc. (add k Pi for general solution)

```
(* Problem 3*)
```

```
PolarPlot[{2 Sin[t], Sin[t] + Cos[t]}, {t, 0, 2 Pi}]
```



Find the intersection points by setting the equations equal to each other. The t-value is Pi/4. From t=0 to t=Pi/4, we use the blue curve as the boundary. From t=Pi/4 to t=3Pi/4 we use the yellow curve as the boundary.

```
Area1 = 0.5 Integrate[(2 Sin[t])^2, {t, 0, Pi / 4}]
```

```
0.285398
```

```
Area2 = 0.5 * Integrate[(Sin[t] + Cos[t])^2, {t, Pi / 4, 3 Pi / 4}]
```

```
0.785398
```

```
Area1 + Area2
```

```
1.0708
```

The precise value of the total area is (Pi - 1)/2.

```
(* Problem 4 *)
```

Consider a Comparison Test with SIGMA(e / n^2). This can also be done with the Integral Test. Does the series converge or diverge?

```
(* Problem 5 *)
```

Get a representation for -1 / (4 - x) as a geometric series. Note that the antiderivate of this expression is equal to the given function f(x).
Before integrating, rewrite as -1 / [4 (1- x/4)]. So the ratio r of this geometric series is (x/4). This means the radius is equal to 4. When to integrating, remember to add the integration constant + C, and find its value by plugging in the center (x=0). The new representation, by the Theorem on Term by Term integration and differentiation, is the same as the radius of the original series.