Present neatly ~~on separate paper~~. Justify for full credit. No Calculators.

Name ______________________ Score ______ A (30 minutes) **x10**

1)

Find the Taylor series of $f(x) = \sin x$ at $a = \pi/6$.

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2) Find the radius of convergence and interval of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{2^n (x-2)^n}{(n+2)!}$$

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3)

Prove that if the series $\Sigma_{n=1}^{\infty} a_n$ is absolutely convergent, then the series

$$\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right) a_n$$

is also absolutely convergent.

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4) Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n}$$

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5) Find the sum of the series.

$$\sum_{n=1}^{\infty} [\tan^{-1}(n+1) - \tan^{-1} n]$$

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6) Find a power series representation for the function and determine the radius of convergence.

$$f(x) = x^2 \tan^{-1}(x^3)$$

Present neatly ~~on separate paper~~. Justify for full credit. No Calculators.

Name ______________________ Score ______ F (30 minutes) **x10**

1)

Find the Taylor series of $f(x) = \cos x$ at $a = \pi/3$.

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2) Find the radius of convergence and interval of convergence of the series.

$$\sum_{n=0}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$$

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3)

Prove that if the series $\Sigma_{n=1}^{\infty} a_n$ is absolutely convergent, then the series

$$\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right) a_n$$

is also absolutely convergent.

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4) Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{n}}{n+1}$$

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5) Find the sum of the series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^n}{3^{2n}(2n)!}$$

6) Evaluate the indefinite integral as a power series. What is the radius of convergence?

$$\int \frac{\tan^{-1} x}{x}\, dx$$