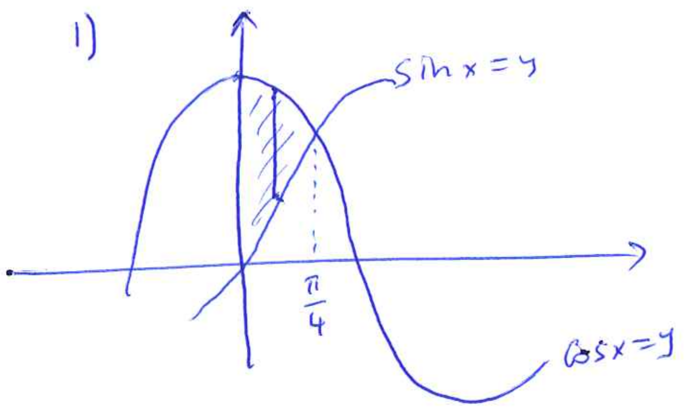
AP Calculus AB Mr. Shubleka Quiz: 47

Present neatly on separate paper. Justify for full credit. No Calculators.

Name KEY / SHUBLEKA Score ______ 12 minutes **Weight: 2**

1) Consider the region enclosed by y = cos x , y = sin x and x = 0 in the first quadrant. Sketch the region neatly and then find:
   a) The area of the region.
   b) The volume of the solid formed when the known cross sections are semi-circles perpendicular to the horizontal axis.
   c) Set up, but do not evaluate, a formula for the volume of the solid formed when the region is revolved about the y-axis.
   d) A vertical line x=k divides the region into two equal parts. Set up, but do not solve, an equation that you would use to find k.

1)



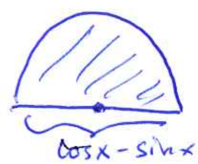
a) $A = \int_0^{\pi/4} \cos x - \sin x \, dx$

$= (\sin x + \cos x)\Big|_0^{\pi/4} =$

$= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - (0 + 1)$

$= \sqrt{2} - 1$ square units.

b)



$r = \frac{\cos x - \sin x}{2}$

$A(x) = \frac{\pi r^2}{2} = \frac{\pi}{2}\left(\frac{\cos x - \sin x}{2}\right)^2$

$A(x) = \frac{\pi}{8}(\cos^2 x + \sin^2 x - \sin 2x)$

$= \frac{\pi}{8}(1 - \sin 2x)$

$V = \int_0^{\pi/4} \frac{\pi}{8}(1 - \sin 2x)\, dx$

$\frac{\pi}{8}\int_0^{\pi/4} 1 - \sin 2x \, dx$

$= \frac{\pi}{8}\left[x + \frac{1}{2}\cos 2x\right]_0^{\pi/4} = \frac{\pi}{8}\left[\left(\frac{\pi}{4} + 0\right) - \left(0 + \frac{1}{2}\right)\right]$

$= \frac{\pi}{8} \cdot \frac{\pi - 2}{4} = \frac{\pi(\pi - 2)}{32}$

c) Shell Method

$V = \int_0^{\pi/4} 2\pi x \cdot (\cos x - \sin x)\, dx$

d) $\int_0^{k} \cos x - \sin x \, dx = \frac{1}{2}(\sqrt{2} - 1)$

or

$\int_0^{k} \cos x - \sin x \, dx = \int_k^{\pi/4} \cos x - \sin x \, dx$