AP Calculus BC　　Mr. Shubleka　　Quiz: 46

Present neatly on separate paper. Justify for full credit. No Calculators.

Name KEY / TAKAYAMA　Score ______ A (30 minutes) **x5**

For questions 1 through 5, determine whether the series converges or diverges.

1) $\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$ ($a_n$)

vs. $\sum \frac{1}{n} = \sum \frac{n^2}{n^3}$ ($b_n$)

Limit Comparison Test.

$\lim_{n\to\infty} \left(\frac{n^2+1}{n^3+1}\right) / \left(\frac{1}{n}\right) = \lim_{n\to\infty} \frac{(n^2+1)\cdot n}{n^3+1} = 1 > 0$

By Limit Comparison Test, $\sum a_n$ and $\sum b_n$ both diverge

2) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

Alternating Series Test (I) $b_{n+1} \le b_n$ (II) $\lim_{n\to\infty} b_n = 0 \to \lim_{n\to\infty} \frac{1}{\sqrt{n+1}} = 0$

$b_n = \frac{1}{\sqrt{n+1}}$

(I) $\frac{1}{\sqrt{n+1+1}} \le \frac{1}{\sqrt{n+1}} \iff \sqrt{n+1} \le \sqrt{n+2}$ ✓ $(n+2 \ge n+1)$

The series converges.

3) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{3n+1}\right)$

Divergence Test:

$\lim_{n\to\infty} \ln\left[\frac{n}{3n+1}\right] = \ln \frac{1}{3} \ne 0$

Diverges to $-\infty$. $\ln 1/3 = -\ln 3 < 0$.

4) $\sum_{n=1}^{\infty} \frac{n^{2n}}{(1+2n^2)^n}$

$\sum_{n=1}^{\infty} \left(\frac{n^2}{1+2n^2}\right)^n$　$\sqrt[n]{|a_n|} = \frac{n^2}{1+2n^2} \to \frac{1}{2} < 1$

It converges absolutely, so it converges.

5) $\sum_{n=1}^{\infty} \frac{(-5)^{2n}}{n^2 9^n}$

$\sum_{n=1}^{\infty} \frac{(-1)^n 5^{2n}}{n^2 \cdot 9^n}$　Ratio Test

$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{5^{2n+2}}{(n+1)^2 \cdot 9^{n+1}} \cdot \frac{n^2 \cdot 9^n}{5^{2n}}\right| = \left|\frac{25}{9} \cdot \frac{n^2}{(n+1)^2}\right| \to \frac{25}{9} > 1$ It diverges

6)

State the following.

(a) The Test for Divergence

(b) The Integral Test

(c) The Comparison Test

a) $\sum a_n$ : If $\lim_{n\to\infty} a_n \to \ne 0$ or DNE, then $\sum a_n$ diverges.

b) If $f(x)$ positive, cont., and eventually decreasing, then $\sum_{n=1}^{\infty} f(n)$ and $\int_1^{\infty} f(x)\,dx$ both converge or both diverge.

c) $\sum a_n$, $\sum b_n$ with $a_n, b_n > 0$

- If $\sum a_n$ converges and $b_n \le a_n$ for all $n$, then $\sum b_n$ converges
- If $\sum a_n$ diverges and $b_n \ge a_n \ \forall n$, then $\sum b_n$ diverges.

Present neatly on separate paper. Justify for full credit. No Calculators.

Name KEY Score ______ F (30 minutes) **x5**

For questions 1 through 5, determine whether the series converges or diverges.

1) $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$

vs $\sum_{n=1}^{\infty} \frac{n}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ $\quad \frac{n}{n^3+1} \le \frac{n}{n^3} \le \frac{1}{n^2}$

By the Comparison Test, since $\sum \frac{1}{n^2}$ converges, then $\sum \frac{n}{(n^3+1)}$ converges as well.

2) $\sum_{n=1}^{\infty} \frac{n^3}{5^n}$

Ratio Test $\sum a_n$ converges.

$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(n+1)^3}{5^{n+1}} \cdot \frac{5^n}{n^3}\right| = \left|\left(\frac{n+1}{n}\right)^3 \cdot \frac{1}{5}\right| \to \frac{1}{5} < 1$

3) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

Integral Test $f(x) = \frac{1}{x\sqrt{\ln x}} = \frac{1}{x}(\ln x)^{-1/2}$

f(x) is positive ✓

continuous ✓

eventually decreasing

{Could Use $f'(x)$}

As $n \uparrow$, $n\sqrt{\ln n} \uparrow$, so $\frac{1}{n\sqrt{\ln n}} \downarrow$ ✓

$\int_2^{\infty} \frac{1}{x\sqrt{\ln x}}\,dx = \lim_{b\to\infty} 2\sqrt{\ln x}\Big|_2^{b} = \infty$

It diverges.

4) $\sum_{n=1}^{\infty} \frac{\cos 3n}{1+(1.2)^n}$

$\sum_{n=1}^{\infty} \left|\frac{\cos 3n}{1+1.2^n}\right|$ $\quad \left|\frac{\cos 3n}{1+1.2^n}\right| \le \frac{1}{1+1.2^n} \le \frac{1}{1.2^n} = \left(\frac{5}{6}\right)^n$

By the Comparison Test, the given series converges absolutely; therefore it converges

5) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{n}}{n+1}$

Alternating Series test. $b_n = \frac{\sqrt{n}}{n+1}$

- $b_{n+1} \le b_n$ $\quad \frac{\sqrt{n+1}}{n+1+1} \le \frac{\sqrt{n}}{n+1}$
- $\lim_{n\to\infty} \frac{\sqrt{n}}{n+1} = \lim_{n\to\infty} \frac{1}{\sqrt{n}+\frac{1}{\sqrt{n}}} = 0$

6)

When a series converges absolutely, it converges.

a) What is an absolutely convergent series? What can you say about such a series?

$\sum a_n$ is absolutely convergent if $\sum |a_n|$ converges.

b) What is a $p$-series? Under what circumstances is it convergent?

Anything like $\sum \frac{1}{n^p}$ For $P > 1$, it converges

c) What is a geometric series? Under what circumstances is it convergent? What is its sum?

$\sum_{n=1}^{\infty} ar^{n-1}$ It converges to $\frac{a}{1-r}$ when $|r| < 1$.