Present neatly ~~on separate paper~~. **Justify** for full credit. No Calculators.

Name KEY/SHUBLEKA Score ______ A (6 minutes) ~~x1~~ 2

1) Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

$$a_n = ne^{-n}$$

2) True or False: "If a sequence converges absolutely, then it converges." Explain.

1) $a_n = n \cdot e^{-n} = \dfrac{n}{e^n}$ $\qquad \dfrac{1}{e}, \dfrac{2}{e^2}, \dfrac{3}{e^3}, \ldots, \dfrac{n}{e^n}, \ldots$

$f(x) = \dfrac{x}{e^x} \qquad x \geq 1$

$f'(x) = \dfrac{e^x \cdot 1 - e^x x}{(e^x)^2} = \dfrac{e^x(1-x)}{e^{2x}} \qquad 1-x \leq 0$ for all $x \geq 1$, $\quad e^{2x} > 0$ for all $x \geq 1$

Therefore, $f'(x) < 0$ in the domain, so $\{f(n)\}$ is decreasing and monotonic. $\quad 0 < \dfrac{n}{e^n} \leq \dfrac{1}{e} \Rightarrow$ Hence bounded.

2) FALSE. CONSIDER $\{(-1)^n\}_{n=1}^{\infty}$.

Present neatly ~~on separate paper~~. **Justify** for full credit. No Calculators.

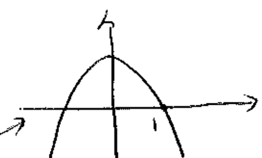
Name KEY / SHUBLEKA Score ______ F (6 minutes) x~~1~~ 2

1) Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

$$a_n = \frac{n}{n^2+1}$$

2) True or False? "If a sequence is bounded, then it converges." Explain.

1) $f(x) = \frac{x}{x^2+1} \quad x \geq 1$



$$f'(x) = \frac{(x^2+1)\cdot 1 - (2x)x}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

negative

$(x^2+1)^2 \rightarrow$ always positive

$f'(x) \leq 0$ for $x \geq 1$. $\{f(n)\}$ is decreasing and monotonic

$0 < \frac{n}{n^2+1} \leq \frac{1}{2}$ for all $n \geq 1$ Hence it is bounded.

2) FALSE. CONSIDER $\{\cos n\}_{n=1}^{\infty}$

$-1 \leq \cos n \leq 1$ for all n.

yet $\{\cos n\}_{n=1}^{\infty}$ diverges.