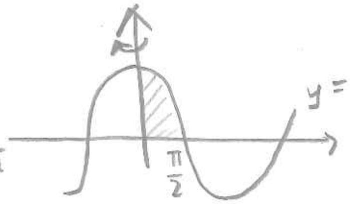
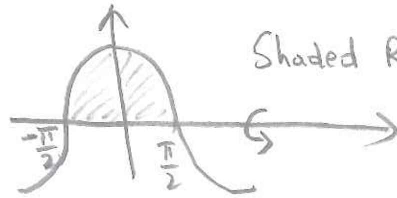
Quiz 45 | AP CALCULUS AB | SOLUTIONS / SHUBLEKA

1)

a) $\int_0^{\pi/2} \underbrace{2\pi x}_{d(x)} \underbrace{\cos x}_{h(x)} \, dx$ Shell Method. Shaded Region Revolved about y-axis.

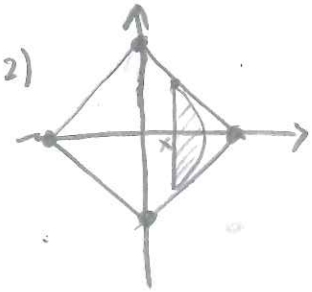


b) $\int_0^{\pi/2} 2\pi \cos^2 x \, dx \longrightarrow = \int_{-\frac{\pi}{2}}^{\pi/2} \pi (\cos x)^2 dx$ Disk Method with symmetry

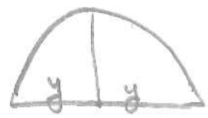


Shaded Region about x-axis.

2)



$V = 2 \int_{x=0}^{x=1} A(x)\, dx$ where $A(x)$ is the area of a typical cross-section.



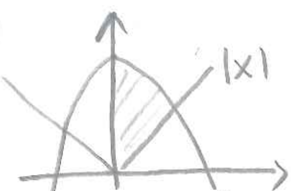
$y = -x+1$

$A = \pi \frac{r^2}{2} = \frac{\pi(1-x)^2}{2}$

$V = 2 \cdot \int_0^1 \frac{\pi}{2}(1-x)^2 dx = \pi \int_0^1 1 - 2x + x^2 dx$

$= \pi \left(x - x^2 + \frac{x^3}{3}\right)\Big|_0^1 = \frac{\pi}{3}$ c.u.

3)



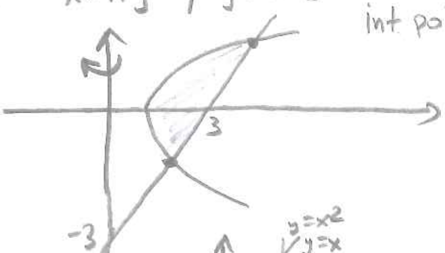
$x = 1 - 2x^2 \Leftrightarrow 2x^2 + x - 1 = 0$

$x = \frac{-1 \pm 3}{4}$ — $1/2$ ✓, -1

$A = 2\int_0^{1/2} 1 - 2x^2 - x \, dx$

$= 2\left(x - \frac{2x^3}{3} - \frac{x^2}{2}\right)\Big|_0^{1/2} = 2\left(\frac{1}{2} - \frac{1}{12} - \frac{1}{8}\right) = \frac{7}{12}$

4) $x = 1+y^2$, $y = x-3$



int points: $1+y^2 = y+3$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

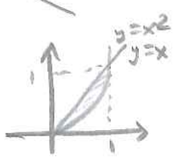
$-1 \le y \le 2$

$V_{WASHER} = \pi \int_{-1}^{2} (y+3)^2 - (1+y^2)^2 dy$

$= \pi \int_{-1}^{2} y^2 + 6y + 9 - 1 - 2y^2 - y^4 dy$

$= \dots = \frac{117\pi}{5}$

5)

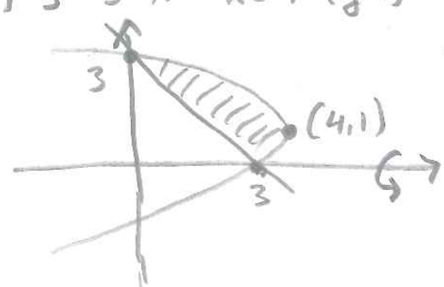


a) $V_{WASHER} = \pi \int_0^1 x^2 - (x^2)^2 dx = \pi\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1 = \frac{2\pi}{15}$ c.u.

b) $V_{SHELL} = 2\pi \int_0^1 x \cdot (x - x^2) dx = 2\pi\left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1 = \frac{\pi}{6}$ c.u.

c) $V_{WASHER} = \pi \int_0^1 (2-x^2)^2 - (2-x)^2 dx = \dots = \frac{8\pi}{15}$

6) $y = 3 - x$ $x = 4 - (y-1)^2$



$V_{SHELL} = 2\pi \int_0^3 y\left[4 - (y-1)^2 - (3-y)\right] dy = \dots = \frac{27\pi}{2}$