Present neatly ~~on separate paper~~. **Justify** for full credit. No Calculators.

Name KEY / SHUBLEKA Score ______ F (6 minutes) **x1**

1) Determine whether the sequence converges or diverges. If it converges, find the limit.

$a_n = \ln(n+1) - \ln n$

2) True or False: "If a sequence converges absolutely, then it converges." Explain.

1) $\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left[\ln(n+1) - \ln n\right]$

$= \lim_{n\to\infty} \ln\left[\frac{n+1}{n}\right] = \ln 1 = 0.$

$\frac{n+1}{n} \to 1$

The sequence converges to zero.

2) False. The correct version is

"If it converges to 0 absolutely, then it converges to zero".

If $|a_n| \longrightarrow 0$, then $a_n \longrightarrow 0$.

Ex. $(-1)^n$ converges absolutely to 1, yet the given sequence diverges.

AP Calculus BC Mr. Shubleka Quiz: 44

Present neatly ~~on separate paper~~. **Justify** for full credit. No Calculators.

Name KEY/SHUBLEKA Score ______ A (6 minutes) **x1**

1) Determine whether the sequence converges or diverges. If it converges, find the limit.

$a_n = n\sin(1/n)$

2) True or False? "If a sequence is bounded, then it converges." Explain.

$\infty \cdot 0$ L'Hôspital's Rule.

1) $a_n = n\sin(1/n)$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \overset{\to\infty}{n} \cdot \overset{\to 0}{\sin(1/n)} = \lim_{n\to\infty} \frac{\sin(1/n)}{1/n}$$

vs. $$\lim_{x\to\infty} \frac{\sin(1/x) \to 0}{1/x \to 0} = \lim_{x\to\infty} \frac{\cos(1/x)\left(\frac{-1}{x^2}\right)}{-1/x^2} = \lim_{x\to\infty} \cos\left(\frac{1}{x}\right)$$

$$= \lim_{x\to\infty} \cos(0^+) = 1.$$

The sequence $\{a_n\}$ converges to 1. $\therefore$

2) False. The correct theorem says: $\left\{\begin{array}{l}\text{Bounded} \\ \text{and} \\ \text{Monotonic}\end{array}\right. \Rightarrow$ Converges

Ex. $a_n = (-1)^n$ is bounded by 1 and -1.

... yet it diverges!!