AP Calculus AB Mr. Shubleka Quiz: 43

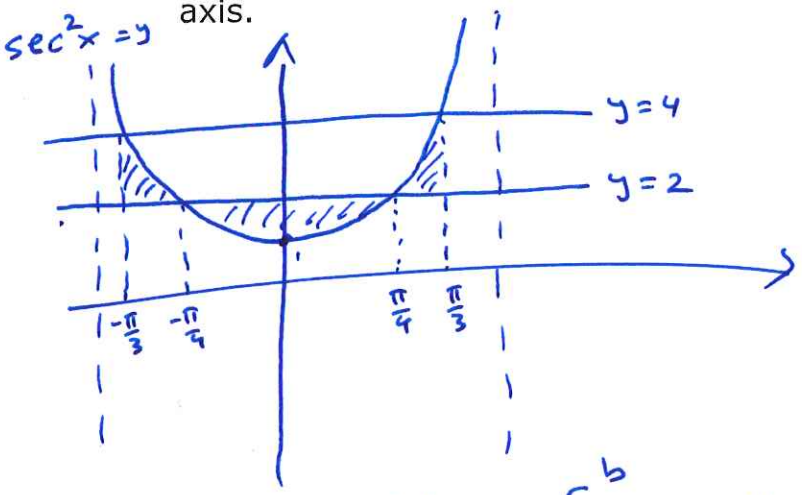
Present neatly on separate paper. Justify for full credit. No Calculators.
Name KEY / SHUBLEKA Score _____ 6 minutes **Weight: x2**

Consider the region enclosed by the curves with the given equations:

$y = \sec^2 x, \quad y = 2, \quad |x| < \frac{\pi}{3}.$

a) Set up and evaluate the area of the described region. Sketch the region neatly.

b) Set up, but do not evaluate, the volume of the solid that results when the region enclosed by the given curves is revolved about the x-axis.



$\sec^2 x = 4$

$\sec x = 2$

$\cos x = 1/2 \rightarrow x = \pm \frac{\pi}{3}$

$\sec^2 x = 2$

$\sec x = \sqrt{2}$

$\cos x = \frac{\sqrt{2}}{2} \rightarrow x = \pm \frac{\pi}{4}$

a) $\text{Area} = \int_a^b y_{above} - y_{below}\, dx$

$= 2\int_0^{\pi/4} 2 - \sec^2 x\, dx + 2\int_{\frac{\pi}{4}}^{\pi/3} \sec^2 x - 2\, dx$

$= 2\,(2x - \tan x)\Big|_0^{\pi/4} + 2\,(\tan x - 2x)\Big|_{\frac{\pi}{4}}^{\pi/3}$

$= 2\left(\frac{\pi}{2} - 1\right) + 2\left[\left(\sqrt{3} - 2\frac{\pi}{3}\right) - \left(1 - \frac{\pi}{2}\right)\right]$

$= \pi - 2 + 2\sqrt{3} - \frac{4\pi}{3} - 2 + \pi = 2\sqrt{3} + \frac{2\pi}{3} - 4$

b) $\text{Volume} = V_{WASHER} =$

$= \pi \cdot 2\int_0^{\pi/4} 2^2 - (\sec^2 x)^2\, dx + \pi \cdot 2 \cdot \int_{\pi/4}^{\pi/3} (\sec^2 x)^2 - 2^2\, dx.$