QUIZ 42 | A PERIOD | 2014-2015 | SHUBLEKA.

① $\int_0^4 \frac{x-1}{x^2-4x-5}\,dx$ $\qquad \frac{x-1}{(x-5)(x+1)} = \frac{A}{x+1} + \frac{B}{x-5}$

$x-1 = (A+B)x + B - 5A$ $\qquad \begin{cases} A+B=1 \\ B-5A=-1 \end{cases}$ $\quad 6A = 2 \quad A=\frac{1}{3} \quad B=\frac{2}{3}$

$= \int_0^4 \frac{1/3}{x+1} + \frac{2/3}{x-5}\,dx$

$= \frac{1}{3}\ln|x+1|\Big|_0^4 + \frac{2}{3}\ln|x-5|\Big|_0^4 = \frac{1}{3}(\ln 5 - \ln 1) + \frac{2}{3}(\ln 1 - \ln 5)$

$= \boxed{-\frac{1}{3}\ln 5}$

② $\int e^{x+e^x}\,dx = \int e^x \cdot e^{e^x}\,dx = \int e^u\,du = e^u + C = e^{e^x} + C$

$u = e^x \quad du = e^x dx$

③ $\int_0^\pi t\cos^2 t\,dt = t\left[\frac{1}{2}t + \frac{1}{4}\sin 2t\right]\Big|_0^\pi - \int_0^\pi \frac{1}{2}t + \frac{1}{4}\sin 2t\,dt$

$u = t \qquad u' = 1$

$v' = \cos^2 t \qquad v = \frac{1}{2}t + \frac{1}{4}\sin 2t$

$\cos^2 t = \frac{\cos 2t}{2} + \frac{1}{2}$

$= \pi\left(\frac{\pi}{2} + \frac{1}{4}0\right) - \left(\frac{t^2}{4} - \frac{1}{8}\cos 2t\right)\Big|_0^\pi$

$= \frac{\pi^2}{2} - \left[\left(\frac{\pi^2}{4} - \frac{1}{8}\right) - \left(\frac{0}{4} - \frac{1}{8}\right)\right]$

$= \frac{2\pi^2}{4} - \frac{\pi^2}{4} = \frac{\pi^2}{4}.$

④ $\int \frac{1}{(x-2)(x^2+4)}\,dx$

$\frac{A}{x-2} + \frac{Bx+C}{x^2+4} = \frac{(A+B)x^2 - (2B-C)x + 4A - 2C}{Q(x)} = \frac{0x^2 + 0x + 1}{Q(x)}$

$\begin{cases} 4A - 2C = 1 \\ A + B = 0 \\ C - 2B = 0 \end{cases} \iff \begin{cases} 2A - C = 1/2 \\ B = -A \\ C = 2B = -2A \end{cases} \iff \begin{cases} C = 2A - 1/2 \\ C = -2A \end{cases} \Rightarrow A = 1/8,\ B = -1/8,\ C = -1/4$

$= \int \frac{1/8}{x-2} + \frac{-1/8\,x - 1/4}{x^2+4}\,dx = \frac{1}{8}\ln|x-2| - \frac{1}{16}\int \frac{2x}{x^2+4}\,dx - \frac{1}{4}\int \frac{1}{x^2+4}\,dx$

$= \frac{1}{8}\ln|x-2| - \frac{1}{16}\ln|x^2+4| - \frac{1}{8}\arctan\left(\frac{x}{2}\right) + C$

Quiz 42 | F PERIOD | 2014-2015 | SHUBLEKA.

(1) $\int \frac{x-1}{x^2-4x+5}\,dx = \int \frac{x-1}{x^2-4x+4+1}\,dx = \int \frac{x-1}{(x-2)^2+1}\,dx$

irreducible quadratic

$u = x-2$; $du = dx$; $x = u+2$

$= \int \frac{u+2-1}{u^2+1}\,du = \int \frac{u+1}{u^2+1}\,du$

$= \int \frac{u}{u^2+1}\,du + \int \frac{1}{u^2+1}\,du = \frac{1}{2}\ln|u^2+1| + \arctan(u) + C$

$= \frac{1}{2}\ln|(x-2)^2+1| + \arctan(x-2) + C$

(2) $\int_1^4 \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt = 2\int_1^4 \frac{e^{\sqrt{t}}}{2\sqrt{t}}\,dt = 2\int_1^2 e^u\,du$

$u = \sqrt{t}$ $\quad du = \frac{1}{2\sqrt{t}}\,dt$

$= 2\, e^u \Big|_1^2 = 2(e^2 - e)$

$= 2e(e-1)$.

(3) $\int (x+\sin x)^2 = \int x^2 + 2x\sin x + \sin^2 x\,dx$

$= \frac{x^3}{3} + 2\int x\sin x\,dx + \int \frac{1}{2} - \frac{\cos 2x}{2}\,dx$

$= \frac{x^3}{3} + 2\left[x(-\cos x) - \int -\cos x\,dx\right] + \frac{1}{2}x - \frac{1}{4}\sin 2x$

$= \frac{x^3}{3} - 2x\cos x + 2\sin x + \frac{1}{2}x - \frac{1}{4}\sin 2x + C$.

(4) $\int \frac{x + \arcsin x}{\sqrt{1-x^2}}\,dx = \int \frac{x}{\sqrt{1-x^2}} + \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$

$= \frac{-1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx + \int \frac{1\cdot dx}{\sqrt{1-x^2}}\arcsin x = -\frac{1}{2}\frac{(1-x^2)^{1/2}}{1/2} + \frac{(\arcsin x)^2}{2} + C$

$= -\sqrt{1-x^2} + \frac{(\arcsin x)^2}{2} + C$