

Present neatly on separate paper. Justify for full credit. No Calculators.

Name KEY/SHUBLEKA Score \_\_\_\_\_ 8 minutes **Weight: x2**

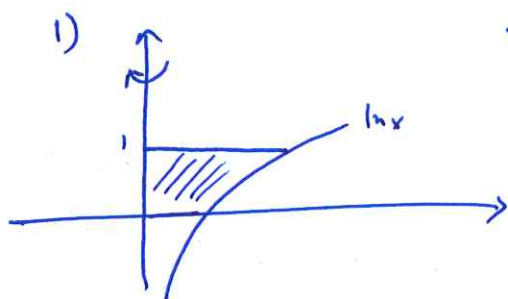
Find the volume of the solid that results when the region enclosed by the given curves is revolved about the y-axis. In each case, sketch the region neatly.

1)

$$y = \ln x, \quad x = 0, \quad y = 0, \quad y = 1$$

2)

$$x = \csc y, \quad y = \pi/4, \quad y = 3\pi/4, \quad x = 0$$

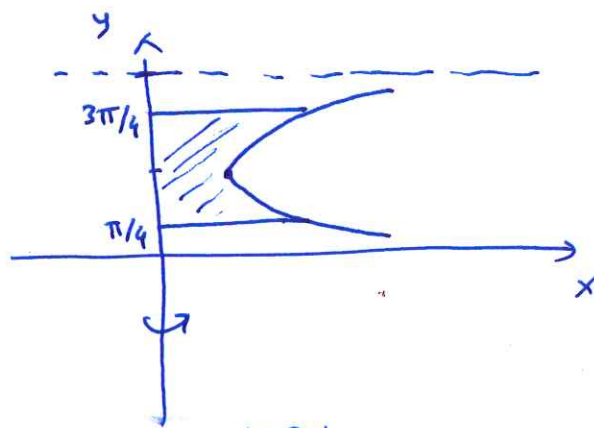
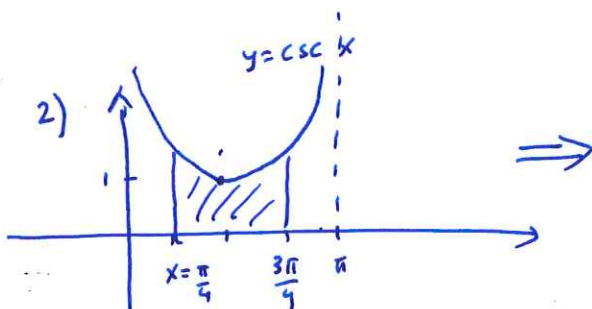


$$V_{\text{Disk}} = \pi \int_{y=0}^{y=1} R^2(y) dy = \pi \int_0^1 (e^y)^2 dy = \pi \int_0^1 e^{2y} dy$$

$$= \frac{\pi}{2} e^{2y} \Big|_0^1 = \frac{\pi}{2} (e^2 - 1)$$

$$R(y) = x = e^y$$

$$y = e^x \leftrightarrow x = \ln y \quad \text{so} \quad y = \ln x \leftrightarrow x = e^y$$



$$V_{\text{Disk}} = \pi \int_{y=\pi/4}^{y=3\pi/4} R^2(y) dy$$

$$x = R(y) = \csc y$$

$$V = \pi \int_{\pi/4}^{3\pi/4} \csc^2 y dy = -\pi \cot y \Big|_{\pi/4}^{3\pi/4}$$

$$= -\pi \left[ \left( \cot \frac{3\pi}{4} \right) - \left( \cot \frac{\pi}{4} \right) \right]$$

$$\text{www.CalculusQuestions.org} = -\pi \left[ -1 - 1 \right] = 2\pi.$$