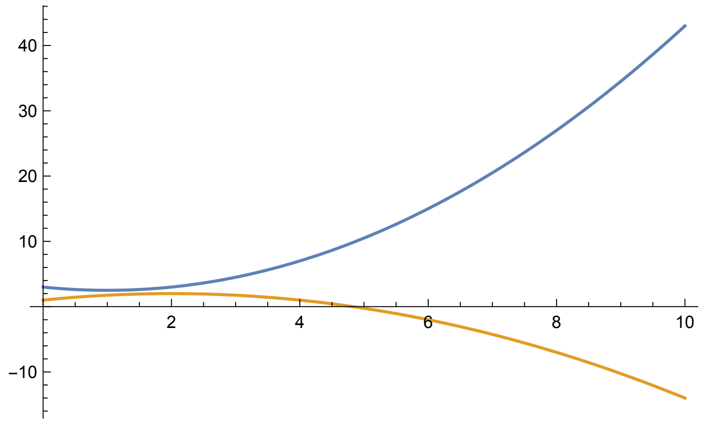
```
(* Problem 1 | Quiz 30 | AP Calculus AB *)
```

In[2]:=
```
s1[t_] := 0.5 t^2 - t + 3;
s2[t_] := -0.25 t^2 + t + 1;
```

In[4]:=
```
Plot[{s1[t], s2[t]}, {t, 0, 10}]
```

Out[4]=

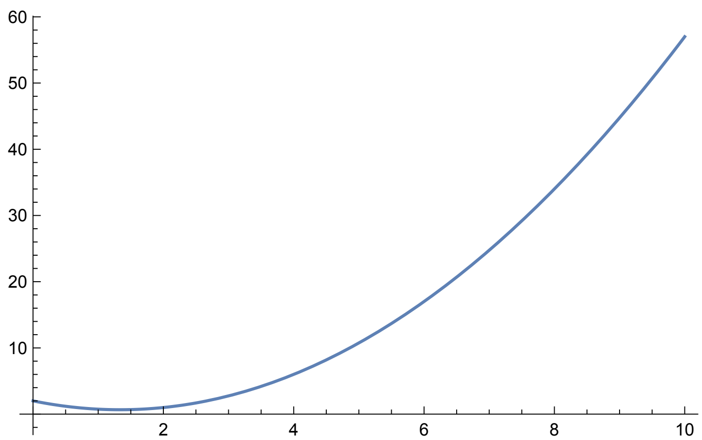



a) To show that the particles do not collide, we need to show that they are not in the same place at the same place. In other words, if we set the position functions s1(t) and s2(t) equal to each-other and try to solve for t, there is no solution.

b) To determine how close the two particles get to each other, consider the "gap" between the position functions above. If we plot s1(t) - s2(t), the minimum value of the difference is the minimum distance between the particles.

In[5]:=
```
Plot[s1[t] - s2[t], {t, 0, 10}]
```

Out[5]=




In[7]:=
```
Solve[s1'[t] - s2'[t] == 0, t]
```

Out[7]=
```
{{t → 1.33333}}
```

The difference s1(t) - s2(t) is at a global minimum when t = 4/3. The respective positions are:

In[8]:=
```
s1[4 / 3]
```

Out[8]=
```
2.55556
```

In[9]:=
```
s2[4 / 3]
```

Out[9]=
```
1.88889
```

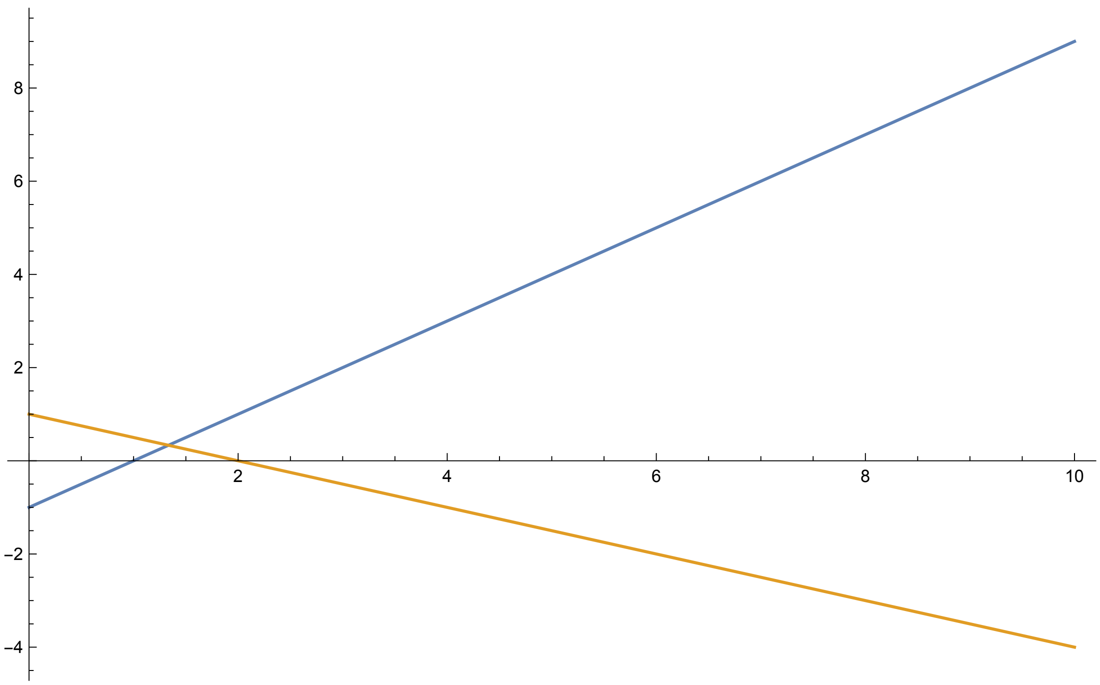
In[10]:=
```
s1[4 / 3] - s2[4 / 3]
```

Out[10]=
```
0.666667
```

The particles get as close as 2/3 of a unit away from each other.
c) To determine the time intervals during which the particles are moving in opposite directions, we find and plot their velocity functions:

In[11]:=
```
Plot[{s1'[t], s2'[t]}, {t, 0, 10}]
```

Out[11]=




The particles' velocities have opposite signs during (0, 1) and (2, infinity). During these time intervals they are moving in opposite directions.

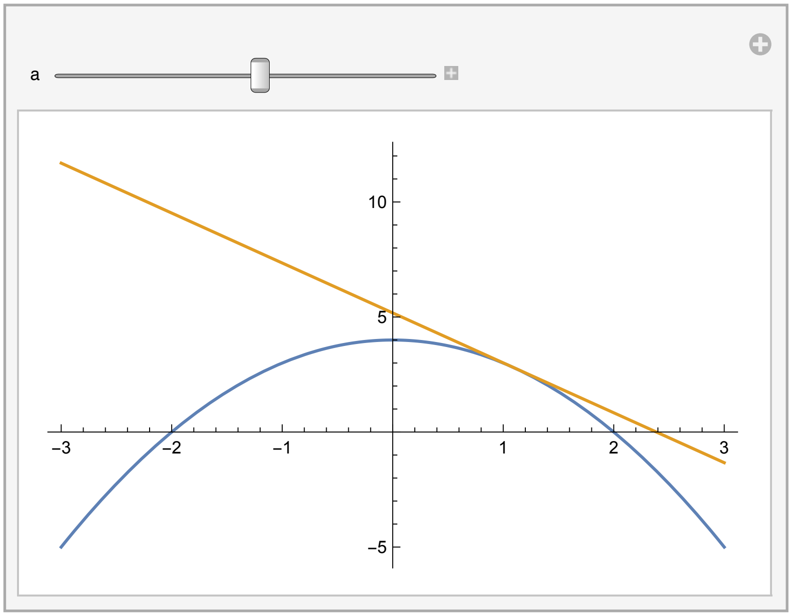
```
(* Problem 2 *)
```

In[12]:=
```
f[x_] := 4 - x^2;
```

At a point (a, 4-a^2), the tangent line has equation y - (4-a^2) = -2 a ( x-a)

In[13]:= `Manipulate[Plot[{4 - x^2, -2 a (x - a) + 4 - a^2}, {x, -3, 3}], {a, 0, 2}]`

Out[13]=

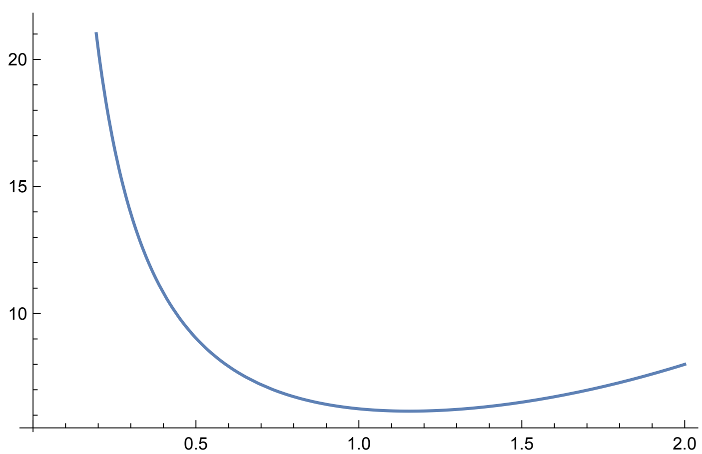



The x and y-intercepts are: (4 + a^2)/ (2a)   and a^2 + 4, respectively. So we consider the area function Q(a) for a between 0 and 2.

In[14]:= `Q[a_] := ((a^2 + 4)^2) / (4 a);`

In[15]:= `Plot[Q[a], {a, 0, 2}]`

Out[15]=




In[16]:= `Solve[Q'[a] == 0, a]`

Out[16]= $\left\{\{a \to -2\,i\},\ \{a \to 2\,i\},\ \left\{a \to -\frac{2}{\sqrt{3}}\right\},\ \left\{a \to \frac{2}{\sqrt{3}}\right\}\right\}$

Justify that the global minimum occurs at  a = 2 / sqrt(3). In the given domain, the derivative changes sign once, from - to +, making Q(2 / sqrt(3)) a global minimum.

In[17]:=
```
Q[2 / Sqrt[3]]
```

Out[17]= $\frac{32}{3\sqrt{3}}$

```
(* Problem 3 *)
```

Proof by contradiction. Assume there are three roots x1, x2, and x3, so that f(x1) = f(x2) =f(x3)= 0. Since f(x) is a polynomial, it is differentiable and continuous on any interval. More specifically, it is continuous on [x1, x2], [x2, x3] and differentiable on (x1, x2) and (x2, x3). We apply Rolle's Theorem on these two intervals, to conlude that
f '(x) must be zero twice. If we set and solve f ' (x) = 0, we determine that there is exactly one real root. A contradiction has been reached, so our assumption that there are three roots is false. QED.