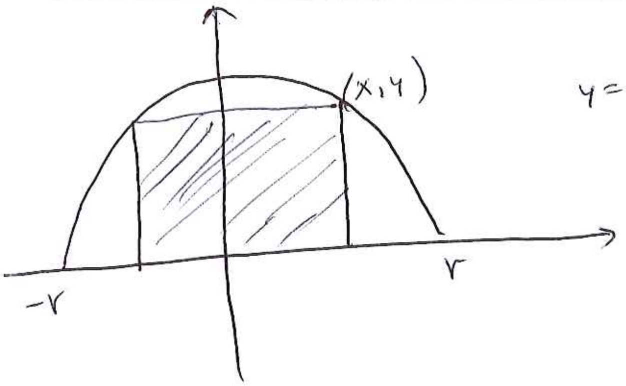
AP Calculus AB Mr. Shubleka Quiz: 27

Present neatly on ~~separate paper~~. Justify for full credit. ~~No Calculators.~~

Name SHUBLEKA / KEY Score ______ 8 minutes

1.

Find the area of the largest rectangle that can be inscribed in a semicircle of radius r.



$y=\sqrt{r^2-x^2}$

$Q = 2\cdot xy$

$Q(x) = 2x\sqrt{r^2-x^2}$

$0 \le x \le r$

$Q(0) = 0 = Q(r)$

$Q'(x) = 2\sqrt{r^2-x^2} + 2x\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{r^2-x^2}}(-2x)$

$= 2\sqrt{r^2-x^2} - \frac{2x^2}{\sqrt{r^2-x^2}}$

$= 2\frac{[r^2-x^2-x^2]}{\sqrt{r^2-x^2}} = \frac{2(r^2-2x^2)}{\sqrt{r^2-x^2}}$ 0 ?

$x^* = \pm\sqrt{\frac{r^2}{2}} = \pm\frac{\sqrt{2}}{2}r$

$x^* = \frac{\sqrt{2}}{2}r$

$y=\sqrt{r^2-x^2}$

$y=\sqrt{\frac{r^2}{2}} = \frac{\sqrt{2}}{2}r$

$Q(x^*) = 2xy = 2\cdot\frac{\sqrt{2}}{2}r\cdot\frac{\sqrt{2}}{2}r = r^2$

By the Closed Interval Method,
the largest area occurs when $x=y=\frac{r}{\sqrt{2}}$, and is equal to r^2.