

(* Quiz 26 | AP Calculus BC | Shubleka *)

(* Problem 1 | A Period *)

In[7]:= $f[x_] := (a^x) / (1 + a^{(x+k)})$;

In[8]:= **Simplify**[f' [x]]

Out[8]=
$$\frac{a^x \text{Log}[a]}{(1 + a^{k+x})^2}$$

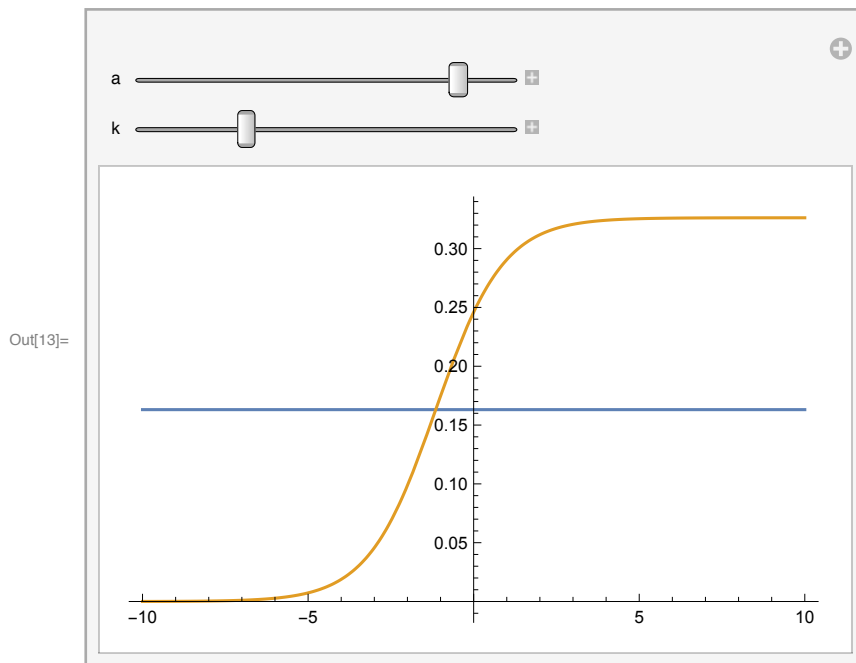
In[9]:= **Simplify**[f'' [x]]

Out[9]=
$$-\frac{a^x (-1 + a^{k+x}) \text{Log}[a]^2}{(1 + a^{k+x})^3}$$

The second derivative changes sign when $x + k = 0$, or equivalently, when $x = -k$.

Note that the first derivative does not change sign and the original function is continuous everywhere.

In[13]:= **Manipulate**[Plot[{(a^-k) / (1 + a^(-k+k)), (a^x) / (1 + a^(x+k))}, {x, -10, 10}], {a, 0.1, 3}, {k, 0.1, 4}]



(* Problem 1 | F Period *)

Differentiating both sides with respect to x , we get:

$$\cos(x) - \sin(y) y' = 2y'$$

$$\cos(x) = y'(2 + \sin(y))$$

$$dy/dx = (\cos x) / (2 + \sin y)$$

As suggested by the problem, when $\cos x = 0$ we have a critical point.

The second derivative is:

$$y''(x) = [(2 + \sin y)(-\sin x) - (\cos x)(\cos y) y'] / (2 + \sin y)^2$$

When $\cos x = 0$, the second derivative becomes:

$$y''(x) = [(2 + \sin y)(-\sin x)] / (2 + \sin y)^2$$

at $x = \pi/2 + 2k\pi$, $y''(x) < 0$, so we have a relative minimum.

at $x = 3\pi/2 + 2k\pi$, $y''(x) > 0$, so we have a relative maximum.

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In[19]:= ContourPlot[{2 * y == Cos[y] + Sin[x], x == Pi / 2, x == 3 Pi / 2},
  {x, -2 Pi, 2 Pi}, {y, -1, 2}, Axes -> True]
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