AP Calculus AB Mr. Shubleka Quiz: 26

Present neatly on ~~separate paper~~. Justify for full credit. ~~No Calculators.~~

Name KEY / SHUBLEKA Score ______ 15 minutes

1.

Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

2.

Find the point on the hyperbola $xy = 8$ that is closest to the point $(3, 0)$.

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① $x, y > 0$

$x + 4y = 1000 \rightarrow x = 1000 - 4y$

$Q = xy = (1000 - 4y)\,y = 1000y - 4y^2$

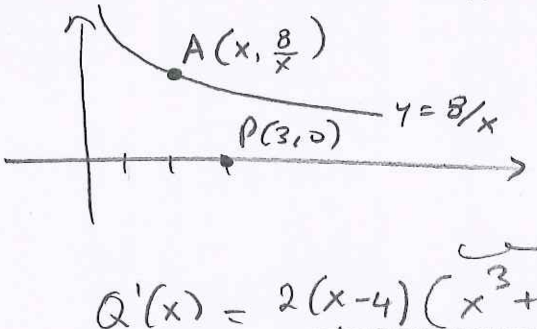
$Q'(y) = 1000 - 8y \overset{?}{=} 0$ $\quad y = \frac{1000}{8} = 125$

$(500, 125) = x, y$ $\quad x = 1000 - 4 \cdot 125 = 500$

Justify: $Q'(y) = 1000 - 8y$ $\quad$ + $y = 125$ − $\quad$ Global Max @ $y = 125$

Alternatively, use Second Derivative Test.

② $xy = 8 \Rightarrow y = 8/x$



$Q = \text{Distance}^2 = (x-3)^2 + \left(\frac{8}{x} - 0\right)^2$

$Q = (x-3)^2 + \frac{64}{x^2}$

$Q'(x) = 2(x-3) - \frac{128}{x^3}$

$= 2x - 6 - \frac{128}{x^3} \overset{?}{=} 0$

$= \frac{2x^4 - 6x^3 - 128}{x^3} \overset{?}{=} 0$

$= \frac{2(x^4 - 3x^3 - 64)}{x^3} \overset{?}{=} 0$

$Q'(x) = \frac{2(x-4)\underbrace{(x^3 + x^2 + 4x + 16)}_{\text{Always positive when } x>0}}{x^3}$ ← Always positive when $x > 0$.

− | +
4

local and global min

Point. $(4, 2)$

CALCULATOR → @ $x = 4$ $Q'(x)$ changes sign.