```
(* Quiz 25 | AP Calculus BC | F | Problem 1 *)
```

In[2]:=
```
f[x_] := (x - 1) / x^2;
```

In[3]:=
```
Simplify[f'[x]]
```

Out[3]= $\frac{2 - x}{x^3}$

In[4]:=
```
Simplify[f''[x]]
```

Out[4]= $\frac{2\ (-3 + x)}{x^4}$

In[5]:=
```
f'[1.9]
```

Out[5]= 0.0145794

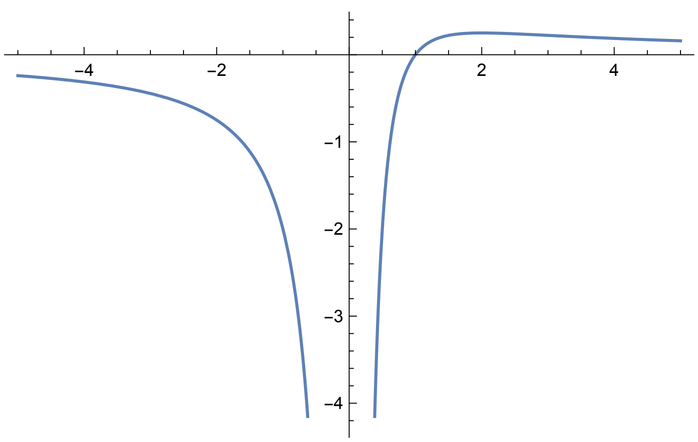
In[6]:=
```
f'[2.1]
```

Out[6]= -0.010798

Local Maximum @ (2, 0.25)
Increasing on (0, 2). Decreasing on (-infinity, 0) and (2, infinity)
Concave down on (-infinity, 0) and (0, 3). Concave up on (3, infinity).
Inflection point at (3, 2/9).
Vertical asymptote: x=0.
Horizontal Asymptote: y=0

In[3]:=
```
Plot[f[x], {x, -5, 5}]
```

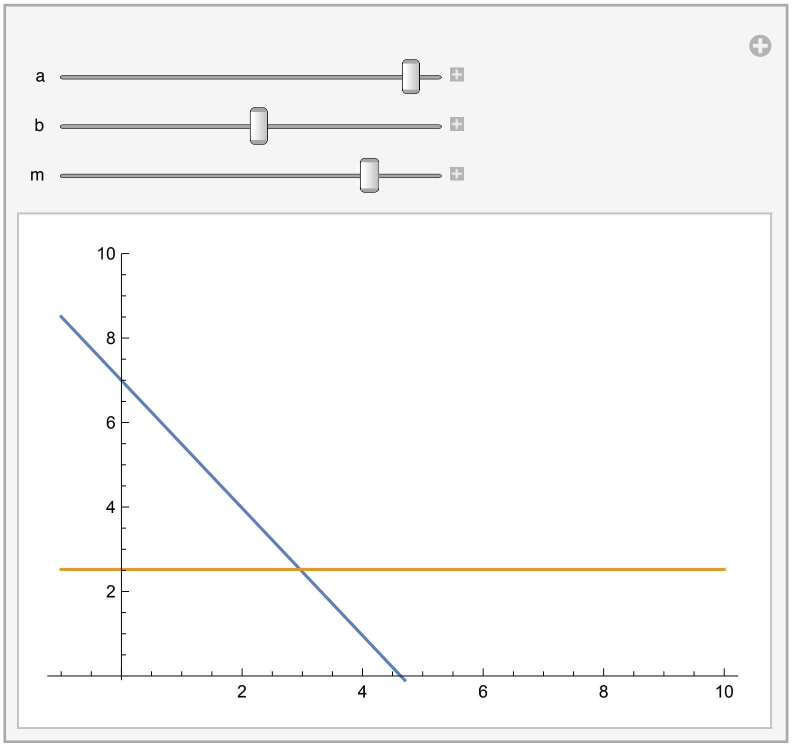
Out[3]=




```
(* Quiz 25 | AP Calculus BC | F | Problem 2 *)
```

In[7]:=
```
Manipulate[Plot[{ m (x - a) + b, b}, {x, -1, 10}, PlotRange → {-0.1, 10}],
 {a, 2, 3}, {b, 2, 3}, {m, -8, -.3}]
```

Out[7]=



In[8]:=
```
Q[m_] := (b - m a) ^2 + ((m a - b) / m) ^2;

FullSimplify[Q'[m]]   (* domain: m < 0)
```

Out[13]=

$$\frac{2\,(-b + a\,m)\,\left(b + a\,m^3\right)}{m^3}$$

The only critical number that matters is m = CubeRoot(-b/a) because the other critical numbers (m=0 and m=b/a) are non-negative and hence not in the domain. At the given critical number, observe that the sign changes from negative to positive, justifying a local and global minimum of the quantity Q(m). (At the critical number, note that both m^3 and (am - b) are negative; therefore, combined, these two factors have no effect on the sign change.)

$$\begin{aligned}
D_{min} &= \left(a - \frac{b}{\sqrt[3]{-\frac{b}{a}}}\right)^2 + \left(b - a \cdot \sqrt[3]{-\frac{b}{a}}\right)^2 \\
&= (a + a^{1/3}b^{2/3})^2 + (b + a^{2/3}b^{1/3})^2 \\
&= (a^2 + 2a^{4/3}b^{2/3} + a^{2/3}b^{4/3}) + (b^2 + 2a^{2/3}b^{4/3} + a^{4/3}b^{2/3}) \\
&= a^2 + 3a^{4/3}b^{2/3} + 3a^{2/3}b^{4/3} + b^2 \\
&= (a^{2/3})^3 + 3(a^{2/3})^2(b^{2/3}) + 3(a^{2/3})(b^{2/3})^2 + (b^{2/3})^3 \\
&= (a^{2/3} + b^{2/3})^3.
\end{aligned}$$

Thus the distance in question is $(a^{2/3} + b^{2/3})^{3/2}$.

(* Quiz 25 | AP Calculus BC | F | Problem 3 *)

To show that the equation has a most one root, we assume it has two roots x=a and x=b such that -2 < a < b < 2. Since f(x) is a polynomial, it is continuous and differentiable in its domain. In particular, it is continuous on [a, b] and differentiable on (a, b). Since f(a) = 0 = f(b), we can use Rolle's theorem to conclude that at least once between a and b, the derivative of f(x) must equal zero.
f '(x) = 3x^2 - 15 = 0
Solving, we get: x = +- Sqrt(5). Both values are outside the interval (a, b) because they are greter than 2 is absolute value. By contradiction, we have shown that f(x) = 0 has at most one solution in the interval [-2, 2].

(* Quiz 25 | AP Calculus BC | A | Problem 1 *)

```
In[25]:= g[x_] := x / (x^2 + 9);

In[26]:= Simplify[g'[x]]
```

Out[26]= $\frac{9 - x^2}{(9 + x^2)^2}$

```
In[27]:= Simplify[g''[x]]
```

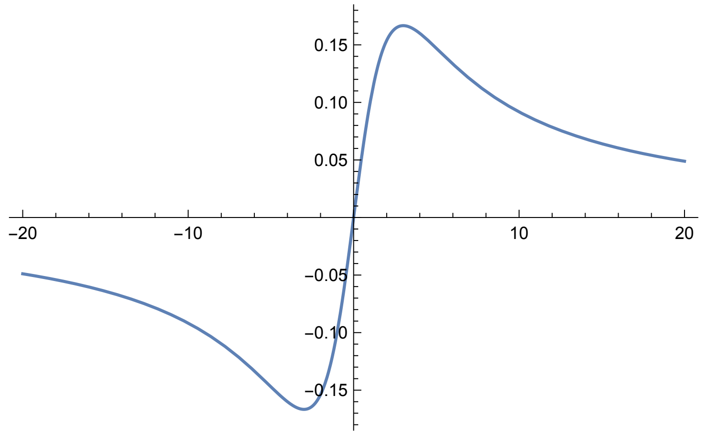
Out[27]= $\frac{2 x (-27 + x^2)}{(9 + x^2)^3}$

Increasing on (-3, 3). Decreasing on (-infinity, -3) and (3, infinity).
Concave down (-infinity, -Sqrt[27]) and (0, Sqrt[27]). Concave up on (-Sqrt[27], 0) and (Sqrt[27], infinity).

Local min at x = -3. Local max at x = 3. Inflection Points at x=0, x = Sqrt[27] and x = - Sqrt[27].
Horizontal Asymptote: y = 0.
Vertical Asymptote: none.

In[30]:=
```
Plot[g[x], {x, -20, 20}]
```

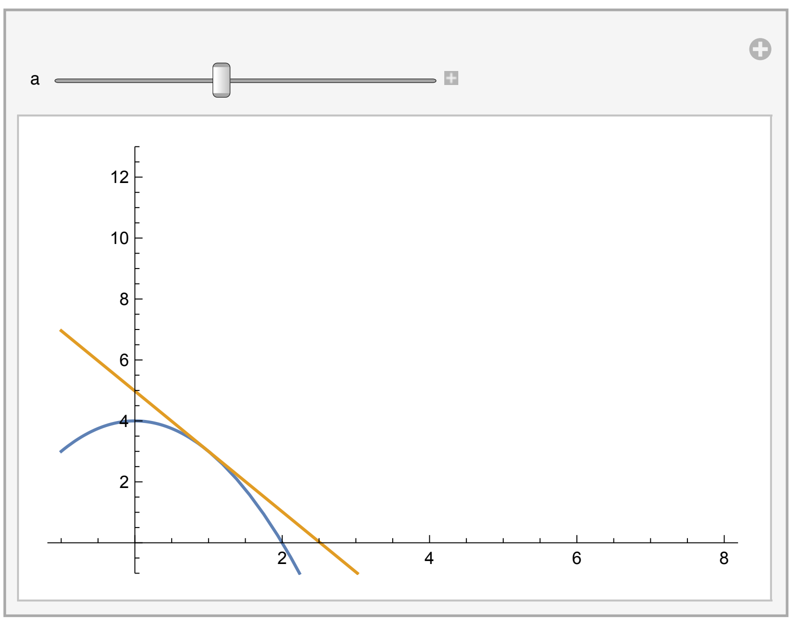
Out[30]=




```
(* Quiz 25 | AP Calculus BC | A | Problem 2 *)
```

In[43]:=
```
Manipulate[Plot[{4 - x^2, (-2 a) (x - a) + (4 - a^2)},
  {x, -1, 8}, PlotRange → {-1, 13}], {a, 0.3, 1.9}]
```

Out[43]=

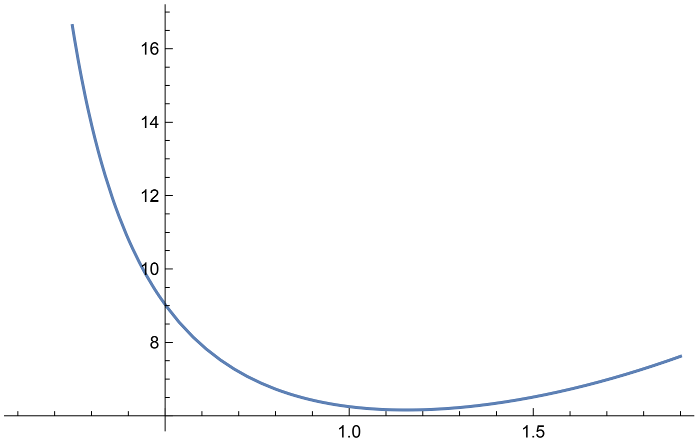



```
(* Area Function =
 A[a]  where a is the x-coordinate of the point where the tangent is drawn. *)
```

In[48]:=
```
A[a_] := 0.5 * (4 + a^2) * (4 + a^2) / (2 a);
```

In[50]:=
```
Plot[A[a], {a, 0.1, 1.9}]
```

Out[50]=



In[46]:=
```
FullSimplify[A'[a]]
```

Out[46]= $2. - \frac{4.}{a^2} + 0.75\, a^2$

In[52]:=
```
2 / Sqrt[3] // N
```

Out[52]= 1.1547

The domain is $0 < a \leq 2$ ($b = 4 - a^2 \geq 0$ forces $a^2 \leq 4$, so $a \leq 2$). Now setting $A'(a) = 0$:

$$0 = A'(a) = -\frac{4}{a^2} + 2 + \frac{3}{4}a^2 = \frac{3a^4 + 8a^2 - 16}{4a^2}.$$

Thus $3a^2 + 8a^2 - 16 = 0$, a quadratic equation in a^2; with the solutions $a^2 = (-8 \pm 16)/6$. a^2 cannot be negative, so the only valid solution is $a^2 = 8/6 = 4/3$, so $a = 2/\sqrt{3}$, which is in the domain. Now $A'(a)$ is seen to be negative near 0, positive for large values of a, thus we have a minimum (single critical point in the interval criterion!). The corresponding area is $\boxed{A = \frac{32\sqrt{3}}{9}}$.

```
(* Quiz 25 | AP Calculus BC | A | Problem 3 *)
```

To show that the equation has at most two real roots, we assume that the function f(x) = x^4 + 4x + c has three zeroes: x=a, x=b, x=c.
Assume that a < b < c.
Since f(x) is continuous and differentiable on any subinterval of its domain (all reals), we can apply Rolle’s Theorem separately on the closed intervals [a,b] and [b, c], since f(a) = f(b) = f(c) = 0.
By Rolle’s Theorem, it follows that there must exist numbers d and e such that a < d < b < e < c and f ‘(d) = 0 = f ‘(e). The first derivative of f(x) is f ’(x) = 4x^3 + 4. If we set it equal to zero, we get one solution (x= -1) and not two as our logic suggests.
Therefore, by contradiction, we have shown that the given polynomial cannot have three zeroes. Hence the equation has at most two roots.