Present neatly on ~~separate paper~~. Justify for full credit. No Calculators.

Name _____ Score _____ 8 minutes / A

1)

Suppose that the position functions of two particles, P_1 and P_2, in motion along the same line are

$$s_1 = \tfrac{1}{2}t^2 - t + 3 \quad \text{and} \quad s_2 = -\tfrac{1}{4}t^2 + t + 1$$

respectively, for $t \geq 0$.

(a) Prove that P_1 and P_2 do not collide.

(b) How close do P_1 and P_2 get to each other?

(c) During what intervals of time are they moving in opposite directions?

(a) $s_1 = s_2$ if they collide, so $\frac{1}{2}t^2 - t + 3 = -\frac{1}{4}t^2 + t + 1$, $\frac{3}{4}t^2 - 2t + 2 = 0$ which has no real solution.

(b) Find the minimum value of $D = |s_1 - s_2| = \left|\frac{3}{4}t^2 - 2t + 2\right|$. From part (a), $\frac{3}{4}t^2 - 2t + 2$ is never zero, and for $t = 0$ it is positive, hence it is always positive, so $D = \frac{3}{4}t^2 - 2t + 2$. $\frac{dD}{dt} = \frac{3}{2}t - 2 = 0$ when $t = \frac{4}{3}$. $\frac{d^2D}{dt^2} > 0$ so D is minimum when $t = \frac{4}{3}$, $D = \frac{2}{3}$.

(c) $v_1 = t - 1$, $v_2 = -\frac{1}{2}t + 1$. $v_1 < 0$ if $0 \leq t < 1$, $v_1 > 0$ if $t > 1$; $v_2 < 0$ if $t > 2$, $v_2 > 0$ if $0 \leq t < 2$. They are moving in opposite directions during the intervals $0 \leq t < 1$ and $t > 2$.

Present neatly on ~~separate paper~~. Justify for full credit. No Calculators.

Name _____ Score _____ 8 minutes / F

Let $s_A = 15t^2 + 10t + 20$ and $s_B = 5t^2 + 40t, t \geq 0,$ be the position functions of cars A and B that are moving along parallel straight lanes of a highway.

(a) How far is car A ahead of car B when $t = 0$?

(b) At what instants of time are the cars next to each other?

(c) At what instant of time do they have the same velocity? Which car is ahead at this instant?

(a) $s_A - s_B = 20 - 0 = 20.$

(b) $s_A = s_B$, $15t^2 + 10t + 20 = 5t^2 + 40t$, $10t^2 - 30t + 20 = 0$, $(t-2)(t-1) = 0$, $t = 1$ or $t = 2$.

(c) $v_A = v_B$, $30t + 10 = 10t + 40$, $20t = 30$, $t = 3/2$. When $t = 3/2$, $s_A = 275/4$ and $s_B = 285/4$ so car B is ahead of car A.