AP Calculus AB Mr. Shubleka Quiz: 24

Present neatly on ~~separate paper~~. Justify for full credit. ~~No Calculators.~~

Name SHUBLEKA/KEY. Score _______ 8 minutes

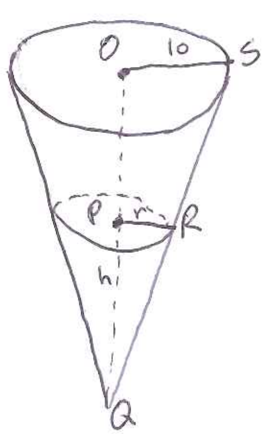
1.

A conical water tank with vertex down has a radius of 10 ft at the top and is 24 ft high. If water flows into the tank at a rate of 20 ft^3/min, how fast is the depth of the water increasing when the water is 16 ft deep? (8 points)

2.

Use an appropriate local linear approximation to estimate the value of cot 46°, and compare your answer to the value obtained with a calculating device. (2 points)

1



$OS = 10$ ft

$PR = r$

$PQ = h$

$QQ = 24$ ft

$Q = \text{volume} = \frac{1}{3}\pi r^2 h$

$\frac{dQ}{dt} = 20$ ft³/min

$\left.\frac{dh}{dt}\right|_{h=16} = ?$

$\frac{h}{24} = \frac{r}{10}$

$\hookrightarrow r = \frac{10h}{24} = \frac{5h}{12} \rightarrow Q = \frac{1}{3}\pi r^2 h$

$Q = \frac{1}{3}\pi\left[\frac{5h}{12}\right]^2 \cdot h$

$Q = \frac{\pi}{3}\cdot\frac{25}{144}\cdot h^3$

$\frac{dQ}{dt} = \frac{\pi}{\cancel{3}}\cdot\frac{25}{144}\cdot \cancel{3}h^2\cdot\frac{dh}{dt}$

$20 = \frac{25\pi}{144}\cdot h^2\cdot\frac{dh}{dt}$ @ $h = 16 \Rightarrow 20 = \frac{25\pi}{144}\cdot 16^2\cdot\frac{dh}{dt}$

$\Rightarrow \frac{20}{25}\cdot\frac{1}{\pi}\cdot\frac{144}{16^2} = \frac{dh}{dt}$

$\frac{4}{5\pi}\cdot\frac{12\cdot 12}{16\cdot 16} = \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{9}{20\pi}$ ft/min

When the water is 16 feet deep, the depth is increasing at a rate of $\frac{9}{20\pi}$ ft/min.

2

$a = 45° = \frac{\pi}{4}$

$b = 46° = \frac{46\cdot\pi}{180} = \frac{23\pi}{90}$

$f(x) = \cot x$

$f'(x) = -\csc^2 x$

$L(x) = f\left(\frac{\pi}{4}\right) + f'\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right)$

$L(x) = 1 - \csc^2\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right)$

$L(x) = 1 - 2\left(x - \frac{\pi}{4}\right)$

$L\left(\frac{23\pi}{90}\right) = 1 - 2\left(\frac{23\pi}{90} - \frac{\pi}{4}\right)$

$\approx 0.965093 \approx \cot 46°$

Device $\rightarrow f(46°) = 0.965689$