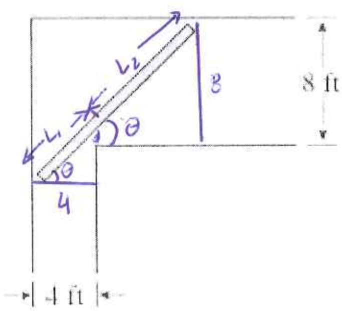
AP Calculus BC　　Mr. Shubleka　　Quiz: 23

Present neatly on ~~separate paper~~. Justify for full credit. ~~No Calculators.~~

Name SHUBLEKA / KEY.　　Score ______ 10 minutes / A

1)

A pipe of negligible diameter is to be carried horizontally around a corner from a hallway 8 ft wide into a hallway 4 ft wide. What is the maximum length that the pipe can have?



$Q = L_1 + L_2$

$L_1 = \frac{4}{\cos\theta}$　　$L_2 = \frac{8}{\sin\theta}$

$Q(\theta) = 4\sec\theta + 8\csc\theta$　　$0 < \theta < \frac{\pi}{2}$

$Q'(\theta) = 4\sec\theta\tan\theta - 8\csc\theta\cot\theta$

$Q'(\theta) = \frac{4\sin\theta}{\cos^2\theta} - \frac{8\cos\theta}{\sin^2\theta} =$

$Q'(\theta) = \frac{4\sin^3\theta - 8\cos^3\theta}{\cos^2\theta\sin^2\theta}$ $\begin{cases} 0 \\ ? \end{cases}$

$4\left[\sin^3\theta - 2\cos^3\theta\right] = 0$

$\sin^3\theta = 2\cos^3\theta$

$\tan^3\theta = 2$

$\tan\theta = \sqrt[3]{2}$

$\theta = \arctan\left(\sqrt[3]{2}\right) = \theta^*$

Note: This is only one possible approach

Use technology to plot $Q(\theta)$ and $Q'(\theta)$ to observe a maximum at $\theta = \arctan(2^{1/3})$ and a sign change of $Q'(\theta)$ from + to −.

(First derivative Test: $\begin{cases} \sin\theta > \sqrt[3]{2}\cos\theta & \text{when } \theta < \theta^* \\ \sin\theta < \sqrt[3]{2}\cos\theta & \text{when } \theta > \theta^* \end{cases}$)

$Q(\theta^*) \approx 16.648$ feet. ← Global Max.

The maximum length that a pipe can have is approximately 16.648 feet.

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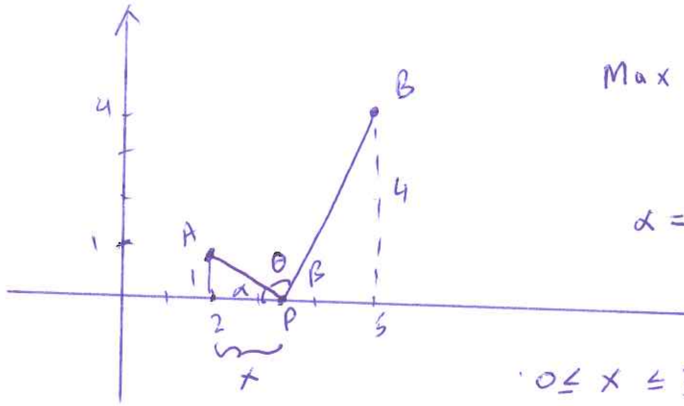
Name Shubleka/Key. Score ______ 10 minutes / F

1)

Given points A(2, 1) and B(5, 4), find the point P in the interval [2, 5] on the x-axis that maximizes angle *APB*.



$\text{Max } \theta \iff \text{Max}(\pi - \alpha - \beta)$

$\iff \text{Min}(\alpha + \beta)$

$\alpha = \arctan\left(\frac{1}{x}\right) \qquad \beta = \arctan\left(\frac{4}{3-x}\right)$

$0 \le x \le 3 \qquad Q = \arctan\left(\frac{1}{x}\right) + \arctan\left(\frac{4}{3-x}\right)$

$$Q'(x) = \frac{1}{1+\left(\frac{1}{x}\right)^2}\cdot\left(-\frac{1}{x^2}\right) + \frac{1}{1+\left(\frac{4}{3-x}\right)^2}\cdot\frac{4}{(3-x)^2} =$$

$$Q'(x) = \frac{-1}{x^2+1} + \frac{4}{(3-x)^2+16} = \frac{-16-(3-x)^2+4(x^2+1)}{(1+x^2)\left(16+(3-x)^2\right)}$$

$$Q'(x) = \frac{-16-9+6x-x^2+4x^2+4}{(1+x^2)(16+9-6x+x^2)} = \frac{-21+3x^2+6x}{(1+x^2)(x^2-6x+25)} =$$

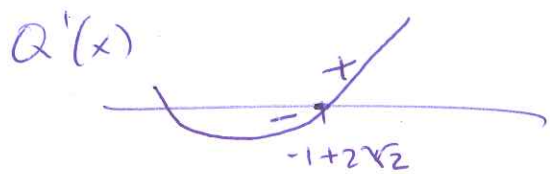
$$Q'(x) = \frac{3\left[x^2+2x-7\right]}{\underbrace{(x^2+1)}_{\text{Always}>0}\underbrace{(x^2-6x+25)}_{\text{Always}>0}} \; \begin{matrix} < 0 \\ > ? \end{matrix}$$

$x^2+2x-7=0$

$x = \frac{-2 \pm \sqrt{32}}{2}$

$x = -1 \pm 2\sqrt{2}$

Only $x = -1+2\sqrt{2}$ is in the domain.



By the First Derivative Test: a local Minimum.

Closed Interval Method:

$Q(0) = \arctan\left(\frac{4}{3}\right) = 0.927 \quad Q(3) = \arctan\left(\frac{1}{3}\right) = 0.322 \quad Q\left(-1+2\sqrt{2}\right) = 1.786 \leftarrow \text{Max } (\alpha+\beta)$