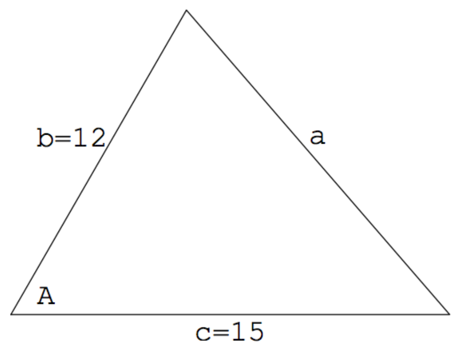
Present neatly on separate paper. Justify for full credit. ~~No Calculators.~~

Name ________________________ Score _______ 15 minutes

1.

Two sides of a triangle have lengths 12 m and 15 m. The angle between them is increasing at a rate of 2°/min. How fast is the length of the third side increasing when the angle between the sides of fixed length is 60°?



Changing degrees to radians: We have $\frac{dA}{dt} = \frac{\pi}{90}$ rad/min when $A = \pi/3$.

We will use the law of cosines with the variables as labeled in the figure,

$$a^2 = b^2 + c^2 - 2bc\cos A.$$

Since $b = 12$ and $c = 15$ are constants, we can plug them in before implicitly differentiating with respect to t.

$$a^2 = 12^2 + 15^2 - 2(12)(15)\cos A$$

$$2a\frac{da}{dt} = -2(12)(15)(-\sin A)\frac{dA}{dt}$$

$$\frac{da}{dt} = \frac{(12)(15)\sin A}{a}\frac{dA}{dt}$$

At the instant we are interested in discussing, we have $b = 12$, $c = 15$, $A = \pi/3$, and, using the law of cosines,

$$a = \sqrt{12^2 + 15^2 - 2(12)(15)\cos(\pi/3)} = \sqrt{189}.$$

So

$$\frac{da}{dt} = \frac{(12m)(15m)\sin(\pi/3)}{\sqrt{189}m}\frac{\pi}{90min} = \frac{(12)(15)(\sqrt{3}/2)\pi}{90\sqrt{189}}m/min = \frac{\sqrt{3}\pi}{\sqrt{189}}m/min \approx 0.395m/min.$$

When the angle between the sides of fixed length is 60 degrees, the length of the third side is increasing at a rate of approximately 0.395 meters per minute.

2.

Two sides of a triangle are 4 m and 5 m in length and the angle between them is increasing at a rate of 0.06 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\pi/3$.

Solution. Let θ be the angle between the sides of length 4 m and 5 m, so that $d\theta/dt = 0.06$. Drawing a perpendicular across from θ shows us that the height of the perpendicular is $h = 4\sin\theta$, and the base of the triangle is $b = 5$. So the area is

$$A = \left(\frac{1}{2}\right) 5 \cdot 4\sin\theta = 10\sin\theta$$

which means that

$$\frac{dA}{dt} = 10\cos\theta\left(\frac{d\theta}{dt}\right).$$

Thus, $\frac{dA}{dt} = 10\cos\left(\frac{\pi}{3}\right)0.06 = 5*0.06 = 0.3\, m^2/s$.