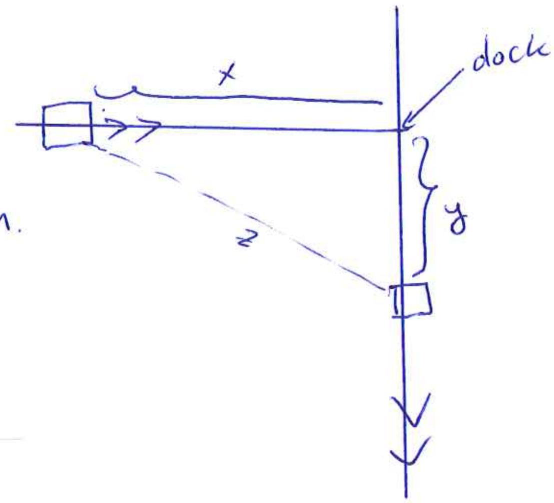
AP Calculus BC Mr. Shubleka Quiz: 22

Present neatly on ~~separate paper~~. Justify for full credit. ~~No Calculators~~.

Name SHUBLEKA | KEY Score ______ 8 minutes / A

1)

A boat leaves a dock at 2:00 PM and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 3:00 PM. At what time were the two boats closest together?



$x(t) = 15 - 15t$

$0 \le t \le 1$ where $t=0$ means $t = 2PM$.

$y(t) = 20t$

$Q = z^2 = (x(t))^2 + (y(t))^2$

$\text{Min}(z) \iff \text{Min}(z^2)$

$Q = (15-15t)^2 + (20t)^2$

$Q'(t) = 2(15-15t)\cdot(-15) + 800t$

$Q'(t) = (30-30t)(-15) + 800t$

$Q'(t) = -450 + 450t + 800t = 1250t - 450$ — 0 ?

$t = \frac{45}{125} = \frac{9}{25}$ hrs

Closed Interval Method (Q is continuous on $[0,1]$)

$Q(0) = 225 \rightarrow z = 15\text{ km} = \sqrt{225}$

$Q(1) = 400 \rightarrow z = 20\text{ km} = \sqrt{400}$

$Q\left(\frac{9}{25}\right) = 144 \rightarrow z = 12\text{ km} = \sqrt{144}$ $\longrightarrow$ Global minimum @ $t = \frac{9}{25}$

$2\text{PM} + \frac{9}{25}$ hrs gives Time = 2:21:36.

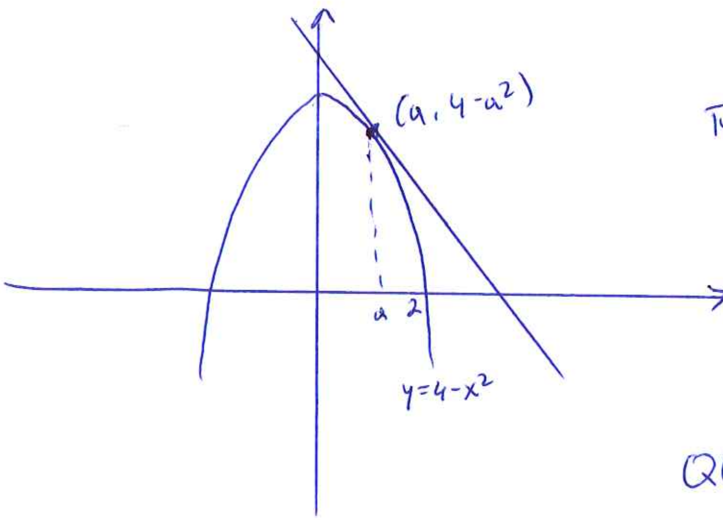
AP Calculus BC Mr. Shubleka Quiz: 22

Present neatly on ~~separate paper~~. Justify for full credit. ~~No Calculators~~.

Name Shubleka / Key. Score ______ 8 minutes / F

1)

What is the smallest possible area of the triangle that is cut off by the first quadrant and whose hypotenuse is tangent to the parabola $y = 4 - x^2$ at some point?



Tangent: $y - (4-a^2) = -2a(x-a)$

$y = -2ax + 2a^2 + 4 - a^2$

$y = -2ax + 4 + a^2$

x-int: $\dfrac{-(4+a^2)}{-2a} = \dfrac{4+a^2}{2a}$

y-int: $4+a^2$

$Q(a) = \frac{1}{2}\text{ base} \cdot \text{height} = \frac{1}{2} \cdot \dfrac{(4+a^2)(4+a^2)}{2a}$

$Q(a) = \dfrac{(4+a^2)^2}{4a} \qquad 0 < a \le 2$

$Q'(a) = \dfrac{4a \cdot 2(4+a^2)\cdot 2a - 4(4+a^2)^2}{(4a)^2}$

$Q'(a) = \dfrac{4\left[4a^2(4+a^2) - (4+a^2)^2\right]}{16a^2}$

$Q'(a) = \dfrac{(4+a^2)\left[4a^2 - (4+a^2)\right]}{4a^2}$

$Q'(a) = \dfrac{(4+a^2)(3a^2-4)}{4a^2}$ 0 ?

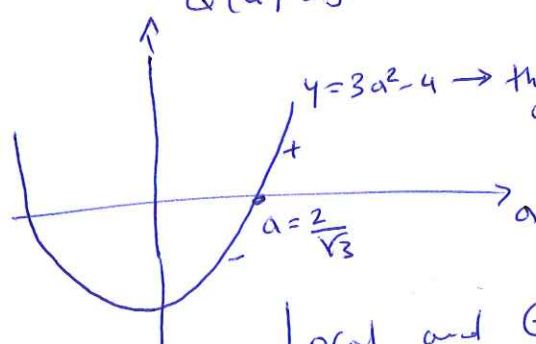
$a = 0$ or $3a^2 - 4 = 0$

not in domain

$a^2 = \frac{4}{3}$

$a = \pm\frac{2}{\sqrt{3}}$

Q'(a) sign



$y = 3a^2 - 4 \rightarrow$ the only factor whose sign matters

Local and Global Min on $(0, 2]$, by the First Derivative Test for Global Extrema.

$Q\left(\frac{2}{\sqrt{3}}\right) = \dfrac{\left(4+\frac{4}{3}\right)^2}{4\sqrt{\frac{4}{3}}} = \dfrac{16^2}{9} \cdot \dfrac{1}{\frac{8}{\sqrt{3}}}$

Minimum Area. $\longrightarrow = \dfrac{32}{3\sqrt{3}}$