AP Calculus BC Mr. Shubleka Quiz: 21

Present neatly on separate paper. Justify for full credit. No Calculators.

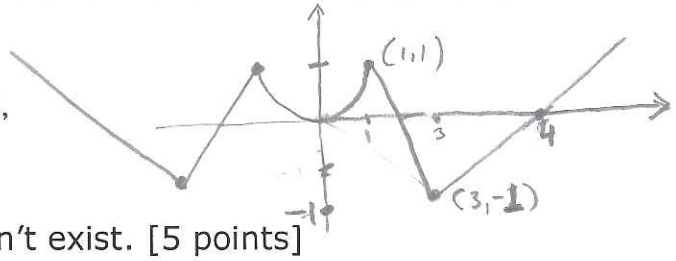
Name SHUBLEKA / KEY Score _____ 15 minutes / A x 4

1) Use the sketching guidelines we learned in class to discuss the function: (20 points)

$$y = \frac{1}{x^2} - \frac{1}{(x-2)^2} = \frac{(x-2)^2 - x^2}{x^2(x-2)^2} = \frac{4-4x}{x^2(x-2)^2} = \frac{4(1-x)}{x^2(x-2)^2}$$

2) Sketch a function that satisfies all the given conditions: (5 points)

$f(0) = 0$, f is continuous and even,
$f'(x) = 2x$ if $0 < x < 1$, $f'(x) = -1$ if $1 < x < 3$,
$f'(x) = 1$ if $x > 3$



3) Find the limit or explain why it doesn't exist. [5 points]

a)

$$\lim_{x\to 0} \frac{x3^x}{3^x - 1} \quad (\to 0) = \lim_{x\to 0} \frac{3^x + x3^x \ln 3}{3^x \cdot \ln 3} \quad (\to \ln 3) = \frac{1}{\ln 3}$$

b)

$$\lim_{x\to 0} \frac{(\cos mx - \cos nx)}{x^2} \quad (\to 0) = \lim_{x\to 0} \frac{-m\sin mx + n\sin nx}{2x} \quad (\to 0) = \lim_{x\to 0} \frac{-m^2\cos mx + n^2\cos nx}{2} \quad (\to n^2 - m^2) = \frac{n^2-m^2}{2}$$

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① Domain $\mathbb{R} - \{0, 2\}$

x-int: $y=0 \to x=1$
y-int: none
VA: $x=0$, $x=2$
HA: $y=0$
No symmetry.

$$f'(x) = \frac{2x^3 - 2(x-2)^3}{x^3(x-2)^3} = \frac{2x^3 - 2x^3 + 12x^2 - 24x + 16}{x^3(x-2)^3} = \frac{4(3x^2 - 6x + 4)}{x^3(x-2)^3}$$

$f'(x)$ table:

x	$-\infty$		0		2		∞
x^3		−	0	+		+	
$(x-2)^3$		−		−	0	+	
$f'(x)$		+	?	−	?	+	
$f(x)$		↗		↘		↗	

$3x^2 - 6x + 4 > 0$ for all x $(b^2 - 4ac < 0)$

No local extrema, since $x=0$, $x=2$ are VA-s.

$f(x)$ table

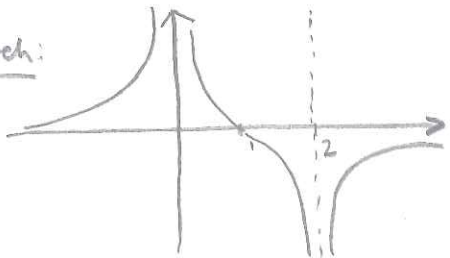
x	$-\infty$		0		1		2		∞
$4(1-x)$		+		+	0	−		−	
x^2		+	0	+		+		+	
$(x-2)^2$		+		+		+	0	+	
$f(x)$		+	?	+	0	−	?	−	
		QII		QI		QIV		QIV	

$$f''(x) = \frac{6}{x^4} - \frac{6}{(x-2)^4} = 6\frac{[(x-2)^4 - x^4]}{x^4(x-2)^4} = \frac{-48(x-1)(x^2-2x+2)}{x^4(x-2)^4}$$

f'': + (∪) | 1 | − (∩) | 2 | +

Inflection Point: $(1, 0)$

Sketch:



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Name KEY / SHUBLEKA. Score _____ 15 minutes / F x 4

1) Use the sketching guidelines we learned in class to discuss the function: (20 points)

$y = x^2/(x+8)$

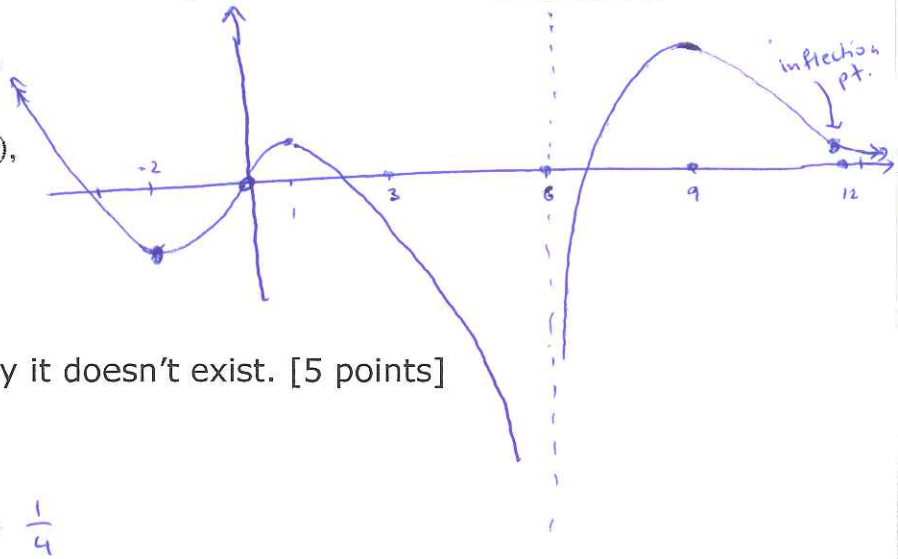
Domain: $x \neq -8$
VA: $x = -8$
HA: none
Slant: $y = x - 8$

x-int: $(0,0)$
y-int: $(0,0)$
Symmetry: None

$f'(x) = x(x+16)/(x+8)^2$
$f''(x) = \frac{128}{(x+8)^3}$

2) Sketch a function that satisfies all the given conditions: (5 points)

$f(0) = 0, \quad f'(-2) = f'(1) = f'(9) = 0,$
$\lim_{x\to\infty} f(x) = 0, \quad \lim_{x\to 6} f(x) = -\infty,$
$f'(x) < 0$ on $(-\infty, -2), (1, 6)$, and $(9, \infty)$,
$f'(x) > 0$ on $(-2, 1)$ and $(6, 9)$,
$f''(x) > 0$ on $(-\infty, 0)$ and $(12, \infty)$,
$f''(x) < 0$ on $(0, 6)$ and $(6, 12)$



3) Find the limit or explain why it doesn't exist. [5 points]

a)

$$\lim_{x\to 0} \frac{x}{\tan^{-1}(4x)} \quad (\to 0/0) = \lim_{x\to 0} \frac{1}{\frac{1\cdot 4}{1+16x^2}} = \frac{1}{4}$$

b)

$$\lim_{x\to 0} \frac{x + \sin x}{x + \cos x} \quad (\to 0+0 \text{ over } 0+1) = \frac{0}{1} = 0.$$

$f'(x)$ table:

x	$-\infty$		-16		-8		0		∞
$(x+8)^2$	+		+		+		+		
x	−		−		−		0		+
$(x+16)$	−		0		+		+		+
$f'(x)$	+		0		−	?	−		0	+
$f(x)$	↗			↘		↘			↗

Local min @ $(0,0)$ Local max @ $(-16,-32)$

$f''(x)$:

	−		+
$f(x)$	∩	−8	∪