AP Calculus AB Mr. Shubleka Quiz: 21

Present neatly on separate paper. Justify for full credit. No Calculators.

Name SHUBLEKA / KEY. Score _______ 15 minutes

1. Find dy/dx. [5 points]

a)

$$y = \frac{1}{\tan^{-1} x} \qquad y = \left(\tan^{-1} x\right)^{-1} \qquad \frac{dy}{dx} = -1 \cdot \left(\tan^{-1} x\right)^{-2} \cdot \frac{1}{1+x^2} = \frac{-1}{(1+x^2)\left[\tan^{-1} x\right]^2}$$

b)

$$y = 2^{\cos x + \ln x} \qquad \frac{dy}{dx} = \ln 2 \quad 2^{\cos x + \ln x} \cdot \left(\frac{1}{x} - \sin x\right)$$

2. Find the limit or explain why it doesn't exist. [5 points]

a)

$$\lim_{\Delta x \to 0} \frac{9\left[\sin^{-1}\left(\frac{\sqrt{3}}{2} + \Delta x\right)\right]^2 - \pi^2}{\Delta x}$$

$$f(x) = 9\left[\sin^{-1} x\right]^2$$

$$x = a = \frac{\sqrt{3}}{2} \qquad f(a) = f\left(\frac{\sqrt{3}}{2}\right) = 9 \cdot \left(\frac{\pi}{3}\right)^2 = \pi^2.$$

$$= \frac{d}{dx}\left[f(x)\right]\Big|_{x=\frac{\sqrt{3}}{2}} = 18 \cdot \sin^{-1} x \quad \frac{1}{\sqrt{1-x^2}}\Big|_{x=\frac{\sqrt{3}}{2}}$$

$$= 18 \cdot \frac{\pi}{3} \cdot \frac{1}{\sqrt{\frac{1}{4}}} = \frac{36\pi}{3} = 12\pi.$$

b)

$$\lim_{h \to 0} \frac{\tan^{-1}(1+h) - \pi/4}{h} = \frac{d}{dx}\left(\tan^{-1} x\right)\Big|_{x=\cancel{\frac{\pi}{4}}\,1}$$

$$= \frac{1}{1+x^2}\Big|_{x=\cancel{\frac{\pi}{4}}\,1} = \frac{1}{1+\cancel{\frac{\pi^2}{16}}\,1} = \frac{\cancel{16}}{\cancel{16+\pi^2}} = \frac{1}{2}$$

3. Find dy/dx. [5 points]

$$\sin^{-1}(xy) = \cos^{-1}(x - y)$$

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(3)

$$\frac{1}{\sqrt{1-x^2y^2}} \cdot (y + xy') = \frac{-1}{\sqrt{1-(x-y)^2}}\,(1 - y')$$

$$y' \frac{x}{\sqrt{1-x^2y^2}} + \frac{y}{\sqrt{1-x^2y^2}} = y' \frac{1}{\sqrt{1-(x-y)^2}} - \frac{1}{\sqrt{1-(x-y)^2}}$$

$$y' \underbrace{\left[\frac{x}{\sqrt{1-x^2-y^2}} - \frac{1}{\sqrt{1-(x-y)^2}}\right]}_{*} = \underbrace{\frac{-1}{\sqrt{1-(x-y)^2}} - \frac{y}{\sqrt{1-x^2y^2}}}_{**}$$

$$\frac{dy}{dx} = \frac{*}{**}$$