AP Calculus BC Mr. Shubleka Quiz: 20

Present neatly. Justify for full credit. ~~No Calculators.~~

Name SHUBLEKA / KEY. Score _____ 15 minutes / A x 2

1) Karen and Oscar start from the same point. Karen walks east at 3 mi/h and Oscar walks northeast at 2 mi/h. How fast is the distance between them changing after 15 minutes?

2) Amy sprints around a circular track of radius 100 m at a constant speed of 7 m/s. Vicky is standing at a distance 200 m from the center of the track. How fast is the distance between Amy and Vicky changing when the distance between them is 200 m?

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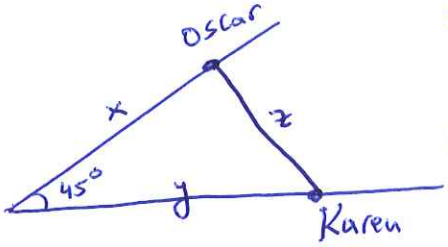
(1)



$z^2 = x^2 + y^2 - 2xy\cos 45^\circ$

$z^2 = x^2 + y^2 - \sqrt{2}\,xy$

$\frac{dx}{dt} = 2$ mph

$\frac{dy}{dt} = 3$ mph

@ $t = 15$ mins : $t = \frac{1}{4}$ hr

x = time · rate $= \frac{1}{4}\cdot 2 = \frac{1}{2} = \frac{2}{4}$

y = time · rate $= \frac{1}{4}\cdot 3 = \frac{3}{4}$

$z^2 = \frac{4}{16} + \frac{9}{16} - \sqrt{2}\cdot\frac{3}{4}\cdot\frac{2}{4} = \frac{13-6\sqrt{2}}{16}$

$z = \frac{\sqrt{13-6\sqrt{2}}}{4}$ miles

$\left.\frac{dz}{dt}\right|_{t=\frac{1}{4}\text{ hr}} = ?$

$2z\cdot\frac{dz}{dt} = 2x\cdot\frac{dx}{dt} + 2y\cdot\frac{dy}{dt} - \sqrt{2}\,(xy)'$

$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt} - \frac{\sqrt{2}}{2}x\frac{dy}{dt} - \frac{\sqrt{2}}{2}y\frac{dx}{dt}}{z} = \frac{\frac{2}{4}\cdot 2 + \frac{3}{4}\cdot 3 - \frac{\sqrt{2}}{2}\cdot\frac{2}{4}\cdot 3 - \frac{\sqrt{2}}{2}\cdot\frac{3}{4}\cdot 2}{\frac{\sqrt{13-6\sqrt{2}}}{4}}$

$\frac{dz}{dt} = \frac{4 + 9 - 6\sqrt{2}/2 - 6\sqrt{2}/2}{\sqrt{13-6\sqrt{2}}} = \frac{13-12\sqrt{2}/2}{\sqrt{13-6\sqrt{2}}} = \frac{13-6\sqrt{2}}{\sqrt{13-6\sqrt{2}}} = \sqrt{13-6\sqrt{2}}$ mph

15 minutes later, the distance between Oscar and Karen is increasing at a rate of $\sqrt{13-6\sqrt{2}}$ miles per hour. ≈ 2.125 mph.

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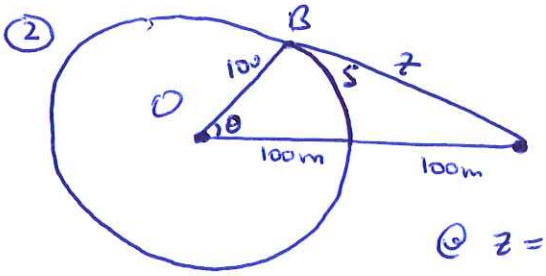
(2)



$s = \theta\cdot r = 100\,\theta$

$\frac{ds}{dt} = 100\cdot\frac{d\theta}{dt} \Rightarrow 7 = 100\cdot\frac{d\theta}{dt} \Rightarrow \boxed{\frac{d\theta}{dt} = \frac{7}{100}}$

$z^2 = 200^2 + 100^2 - 2\cdot 100\cdot 200\cos\theta$

@ $z = 200$: $200^2 = 200^2 + 100^2 - 2000\cdot 200\cos\theta$

$2^2 = 4 + 1 - 4\cos\theta \Rightarrow \cos\theta = \frac{1}{4}$, $\sin\theta = \frac{\sqrt{15}}{4}$

$2z\cdot\frac{dz}{dt} = 40000\sin\theta\cdot\frac{d\theta}{dt}$

$400\frac{dz}{dt} = 40{,}000\cdot\frac{\sqrt{15}}{4}\cdot\frac{7}{100} \Rightarrow \boxed{\frac{dz}{dt} = \frac{7\sqrt{15}}{4}\text{ m/s}}$

When they're 200 m apart, the distance between Amy and Vicky is increasing at a rate of approximately 6.778 m/s.

$\boxed{\frac{dz}{dt} \approx 6.778\text{ m/s}}$

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Name SHUBLEKA/KEY Score _____ 15 minutes / F x 2

1) The minute hand on a watch is 8 mm long and the hour hand is 4 mm long. How fast is the distance between the tips of the hands changing at one o'clock?

2) A lighthouse is located on a small island 3 km away from the nearest point *P* on a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from *P*?

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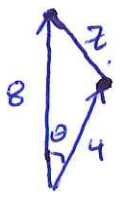
1)



$z^2 = 4^2 + 8^2 - 2\cdot 4 \cdot 8 \cdot \cos\theta$

$z^2 = 16 + 64 - 64\cos\theta$

$z^2 = 80 - 64\cos\theta$

@ 1pm $\theta = \frac{2\pi}{12} = \frac{\pi}{6} \Rightarrow z^2 = 80 - 64\frac{\sqrt{3}}{2} = 80 - 32\sqrt{3}$

$z = \sqrt{80 - 32\sqrt{3}} = 4\sqrt{5 - 2\sqrt{3}}$

$\frac{d\theta}{dt} = \frac{-2\pi}{1} + \frac{2\pi}{12} = \frac{-24\pi + 2\pi}{12} = \frac{-22\pi}{12} = \frac{-11\pi}{6}$ rad/hr

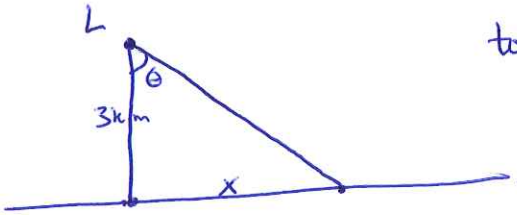
$2z\cdot\frac{dz}{dt} = 64\sin\theta \cdot \frac{d\theta}{dt} \Rightarrow \frac{dz}{dt} = \frac{32\sin\frac{\pi}{6}\left(\frac{-11\pi}{6}\right)}{4\sqrt{5-2\sqrt{3}}} = \frac{4(-11\pi)}{6\sqrt{5-2\sqrt{3}}}$

$\frac{dz}{dt} = \frac{-22\pi}{3\sqrt{5-2\sqrt{3}}} \approx -18.5896$ mm/hr.

or -0.309827 mm/min

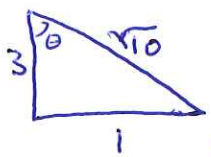
At 1pm the distance between the tips is decreasing at a rate of 18.5896 mm/hr.

2)



$\tan\theta = \frac{x}{3} \Rightarrow x = 3\tan\theta$

$\frac{d\theta}{dt} = 8\pi$ rad/min $\qquad \left.\frac{dx}{dt}\right|_{x=1\text{ km}} = ?$



$\sec\theta = \frac{\sqrt{10}}{3}$

$\sec^2\theta = \frac{10}{9}$

$\frac{dx}{dt} = 3\sec^2\theta \cdot \frac{d\theta}{dt}$

$\frac{dx}{dt} = 3\cdot\frac{10}{9}\cdot 8\pi = \frac{80\pi}{3}$ km/min ≈ 83.776 km/min

When the beam of light is 1 km from point P, it is moving at a rate of 83.776 km/min.