AP Calculus BC Mr. Shubleka Quiz: 18

Present neatly. Justify for full credit. No Calculators.

Name SHUBLEKA | KEY Score ______ ~10 minutes / A x 2

1. If g is the inverse function of $f(x)=2x+\ln x$, find $g'(2)$.

2. Find an equation of the tangent line to the curve $xe^{y}+ye^{x}=1$ at the point (0, 1).

① $f(x)=2x+\ln x$ $g(2) \Rightarrow 2=f(x)=2x+\ln x$ so $\boxed{g(2)=1}$

$g'(x)=\dfrac{1}{f'(g(x))} \Rightarrow$ @ $x=2$ we have $g'(2)=\dfrac{1}{f'(g(2))}=\dfrac{1}{f'(1)}=$

$=\dfrac{1}{\frac{d}{dx}(2x+\ln x)\Big|_{x=1}}=\dfrac{1}{\left(2+\frac{1}{x}\right)\Big|_{x=1}}=\dfrac{1}{2+1}=\dfrac{1}{3}.$

② $\frac{d}{dx}\left(xe^{y}+ye^{x}=1\right)$

$\Leftrightarrow 1\cdot e^{y}+x\cdot e^{y}\cdot\frac{dy}{dx}+\frac{dy}{dx}e^{x}+y\cdot e^{x}=0$

Plug in: (0,1) $e^{1}+0\cdot e^{1}\cdot\frac{dy}{dx}+\frac{dy}{dx}\cdot e^{0}+1\cdot e^{0}=0$

$e+1+\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}\Big|_{(0,1)}=-1-e$

Tangent: $\boxed{y-1=(-1-e)(x-0)}$

OR $\boxed{y=-x-ex+1}$ or $\boxed{y=(-1-e)x+1}$

Present neatly. Justify for full credit. No Calculators.

Name SHUBLEKA/KEY Score _______ ~10 minutes / F x 2

1. If $f(x)=e^x+\ln x$ and $h(x)=f^{-1}(x)$, find $h'(e)$.

2. Let $g(x)=e^{cx}+f(x)$ and $h(x)=e^{kx}f(x)$, where $f(0)=3, f'(0)=5$, and $f''(0)=-2$.

a) Find $g'(0)$ and $g''(0)$ in terms of c.

b) In terms of k, find an equation of the tangent line to the graph of h at the point where $x=0$.

① $f(x)=e^x+\ln x$

$f'(x)=e^x+\frac{1}{x}$

$h(e)=?$

$e=e^x+\ln x \Rightarrow x=1$

So $h(e)=1$

$h'(x)=\dfrac{1}{f'(h(x))}=\dfrac{1}{e^{h(x)}+\frac{1}{h(x)}}$

$h'(e)=\dfrac{1}{e^{h(e)}+\frac{1}{h(e)}}=\dfrac{1}{e^1+\frac{1}{1}}=\dfrac{1}{1+e}$

② a) $g'(x)=ce^{cx}+f'(x)$

$g'(0)=c\cdot e^0+f'(0)=c+5$

$g''(x)=c^2e^{cx}+f''(x)$

$g''(0)=c^2\cdot e^0+f''(0)$

$g''(0)=c^2-2$

b) $h(0)=e^0\cdot f(0)=1\cdot 3=3$

$(0,3)$

$\text{slope}\big|_{(0,3)}=h'(0)=\frac{d}{dx}\left(e^{kx}\cdot f(x)\right)\Big|_{x=0}=\left[ke^{kx}\cdot f(x)+e^{kx}\cdot f'(x)\right]\Big|_{x=0}$

$=k\cdot e^0\cdot f(0)+e^0\cdot f'(0)=k\cdot 3+5=3k+5.$

Tangent: $y-3=(3k+5)x$

$y=(3k+5)x+3.$