AP Calculus BC | Mr. Shubleka | Quiz: 17

Present neatly. Justify for full credit. No Calculators.

Name SHUBLEKA/KEY. Score _______ ~10 minutes / A

1. Find the value of b so that the line $y = x$ is tangent to the graph of $y = \log_b x$. [4 pts]
2. Find the equation of the tangent line to the graph of $y = \ln(5 - x^2)$ at $x = 2$. [4 pts]
3. Find the limit or explain why it does not exist. [2 pts]

$$\lim_{x \to e} \frac{1 - \ln x}{(x - e)\ln x}$$

① $y = x \Rightarrow$ slope $= 1$ $\quad y = \log_b x \quad \frac{dy}{dx} = \left.\frac{1}{x \cdot \ln b}\right|_{x=a} = 1$

$\Leftrightarrow \frac{1}{a \ln b} = 1 \Leftrightarrow a \cdot \ln b = 1$

Also $y = x$ 'meets' $y = \log_b x$ @ $x = a$, so the y-value there is also $y = x = a$. $\Leftrightarrow a = \log_b a \Leftrightarrow a = b^a$

$\begin{cases} a \ln b = 1 \\ a = b^a \end{cases} \Leftrightarrow \begin{cases} \ln b^a = 1 \Rightarrow b^a = e \\ a = b^a = e \Rightarrow (e, e). \end{cases}$

$b^a = e \Leftrightarrow b^e = e \Leftrightarrow b = e^{1/e}.$

② $y = \ln(5 - x^2) \quad \left.\frac{dy}{dx}\right|_{x=2} = \left.\frac{-2x}{5 - x^2}\right|_{x=2} = \frac{-4}{1} = -4$

@ $x = 2$, $y = \ln(5 - 4) = 0$ $\quad$ Tangent: $y - 0 = -4(x - 2)$

$y = -4x + 8$

or $y = 8 - 4x$

③ $\lim_{x \to e} \frac{\frac{1}{\ln x} - \frac{\ln x}{\ln x}}{x - e} = \lim_{x \to e} \frac{(\ln x)^{-1} - 1}{x - e}$

$= \lim_{x \to e} \frac{(\ln x)^{-1} - (\ln e)^{-1}}{x - e} = \left.\frac{d}{dx}\left((\ln x)^{-1}\right)\right|_{x=e} = \left.\left(\frac{-1}{(\ln x)^2} \cdot \frac{1}{x}\right)\right|_{x=e}$

$= \frac{-1}{1} \cdot \frac{1}{e} = \frac{-1}{e}.$

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Name KEY / SHUBLEKA Score ______ ~10 minutes / F

1. Find the value of k for which the graphs of $y=\sqrt{x}+k$ and $y=\ln x$ share a common tangent line at their point of intersection. [4 pts]
2. Find a point on the graph of $y=e^{3x}$ at which the tangent line passes through the origin. [4 pts]
3. Find the limit or explain why it does not exist. [2 pts]

$$\lim_{h\to 0}\frac{(1+h)^{\pi}-1}{h}$$

① $\begin{cases} y=\sqrt{x}+k \\ y=\ln x \end{cases}$ @ $x=a$ they intersect:

$\ln a=\sqrt{a}+k$

and their slopes are the same:

$$\frac{d}{dx}(\sqrt{x}+k)\Big|_{x=a}=\frac{d}{dx}(\ln x)\Big|_{x=a}$$

$$\frac{1}{2\sqrt{a}}=\frac{1}{a}\Rightarrow\frac{a}{2\sqrt{a}}=1$$

$$\Rightarrow \sqrt{a}=2$$

$$\ln 4=\sqrt{4}+k \longleftarrow \boxed{a=4}$$

$$\boxed{\ln 4-2=k}$$

② $y=e^{3x}$ @ (a, e^{3a})

slope $=\frac{dy}{dx}\Big|_{x=a}=3\cdot e^{3a}$

$(0,0)$ to (a, e^{3a}), so

slope is also $\frac{e^{3a}-0}{a-0}=\frac{e^{3a}}{a}$

equal: $3e^{3a}=\frac{e^{3a}}{a}$

$a=\frac{1}{3}$, $f(a)=e^{3\cdot\frac{1}{3}}=e$

point $=\left(\frac{1}{3}, e\right)$

③ $f(x)=x^{\pi}$, $a=1$

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{(1+h)^{\pi}-1^{\pi}}{h}=\frac{d}{dx}\left(x^{\pi}\right)\Big|_{x=1}=\pi\cdot x^{\pi-1}\Big|_{x=1}=\pi$$