

Present neatly on ~~separate paper~~. Justify for full credit. No Calculators.

Name Key / SHUBLEKA Score _____ 10 minutes

1.

Find an equation of the tangent line to the curve
 $y = \tan(\pi x^2/4)$ at the point $(1, 1)$.

2.

Let $r(x) = f(g(h(x)))$, where $h(1) = 2$, $g(2) = 3$, $h'(1) = 4$,
 $g'(2) = 5$, and $f'(3) = 6$. Find $r'(1)$.

3.

If $h(x) = \sqrt{4 + 3f(x)}$, where $f(1) = 7$ and $f'(1) = 4$,
 find $h'(1)$.

$$\textcircled{1} \quad \frac{dy}{dx} = \sec^2\left(\frac{\pi x^2}{4}\right) \cdot \frac{2\pi x}{4} \quad \text{at } x=1 \quad \frac{dy}{dx} = \frac{\pi}{2} \cdot \sec^2\left(\frac{\pi}{4}\right)$$

$$y - 1 = \pi(x - 1)$$

$$= \frac{\pi}{2} \cdot \frac{1}{\cos^2(45^\circ)} =$$

$$= \frac{\pi}{2} \cdot \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = \pi$$

$$\textcircled{2} \quad r'(x) = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$$

$$r'(1) = f'(g(h(1))) \cdot g'(h(1)) \cdot h'(1)$$

$$= f'(g(2)) \cdot g'(2) \cdot 4 = f'(3) \cdot 5 \cdot 4$$

$$= 6 \cdot 5 \cdot 4 = 6 \cdot 20 = 120$$

$$\textcircled{3} \quad h(x) = (4 + 3f(x))^{1/2} \Rightarrow h'(x) = \frac{1}{2} (4 + 3f(x))^{-1/2} \cdot (3f'(x))$$

$$h'(1) = \frac{1}{2} (4 + 3f(1))^{-1/2} \cdot 3 \cdot f'(1) =$$

$$= \frac{1}{2} (4 + 3 \cdot 7)^{-1/2} \cdot 3 \cdot 4 =$$

$$= \frac{1}{2} \cdot \frac{1}{25^{1/2}} \cdot 12 = \frac{6}{5}$$